LITIGATION ECONOMICS DIGEST

Volume III Fall 1998 Number 2

NOTES

The *Litigation Economics Digest (LED)* specializes in presenting applied articles in the discipline of forensic economics, and useful consulting information to the forensic practitioner.

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We are now soliciting articles in forensic economics for publication in future issues. The next issue will be Spring 1999. The *Digest* contains articles in the following arenas: Applied Forensic Economics, Law and Economics, Data Sources, Computer Programs and Ethics in Economics. Those interested should forward two copies of their article, along with any other pertinent information, and a reviewer fee of $25 (payable to LED), to the Editorial Office at the address shown below:

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The National Association of Forensic Economics expresses its thanks to the University of Missouri - Kansas City, for technical support in producing the *Litigation Economics Digest*.

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Thanks from the LED

The editors of the Litigation Economics Digest greatly appreciates the efforts of NAFE members who have devoted their time to reviewing articles during the past three years. The reviewers include the following individuals:

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WHERE WE HAVE BEEN; WHERE WE ARE GOING

This issue of the Litigation Economics Digest (LED) marks the completion of three years of operation. The original goal of the LED was to provide an additional venue for useful articles and information to the subscribers and members of the National Association of Forensic Economics (NAFE). We had hoped for a significant number of quality articles in the areas of commercial damages, environmental damages, antitrust economics, intellectual property issues, use of the Internet, and case studies of interest to our readers. We wanted the LED to be similar to the JFE, yet different in scope and coverage of ideas.

After three years we must conclude we have been only partially successful in this venture. To a large extent the editors of an academic journal are captives of the articles submitted for publication. As a result, somewhat less than half of the published articles over the past three years have been on issues that normally would appear in the JFE. The flip side of this coin is that slightly more than half of our published articles have been non-traditional JFE type articles. So to some extent we have achieved our goal of providing an outlet for ideas that did not traditionally appear in the JFE.

Our goal of publishing case studies has had only limited success. Perhaps this is because our readers may be unclear as to what constitutes a case study. One of our associate editors wrote up an outline for a case study, which appears below.

To be accepted for publication, a case study should meet one or more of the following criteria:

1. Make an economic or legal point that would be of special interest to readers;
2. Demonstrate an unusual or creative solution to a difficult analytic problem, particularly in the area of computing or proving damages; or,
3. Use economic or financial theory to prove (or disprove) negligence in a civil suit or criminal trial.

The case for which the study is being submitted should have special features or interest, but not be so unusual that it would never be encountered again by a forensic economist.

Case studies that are accepted for publication need to meet the same standards for publication as other papers published in the LED. These studies will be sent out for peer review, just as any other article would be handled. Substitution of fictitious names in the case study will be allowed when appropriate. Where a case has already been decided the actual case name should be used. General reference sources used in the case must be cited according to journal standards, but case specific information may be altered to prevent identification where appropriate. In such circumstances, which must be noted in the case study, the editor or a designee, may request to confidentially examine the actual case data to insure that the case characteristics are being correctly presented. The editor or designee will sign a promise of confidentiality and all materials provided to the editor will be returned after examination.

A case study should be formatted as follows:

Case Study: Name (or Dummy Name) of Case
Date Submitted:
Brief Explanation of Special Points or Special Features of the Case

Background of the Case:
Case Analysis:
Conclusion(s); *May suggest needs for research or other general points that could not be made without the reader having the benefit of the Background and Case Analysis provided in the case study.*

A cover sheet should be attached with the name of the author and the case name, but no other information. The submission fee will be the usual $25 charge by the LED.

Robert R. Trout Carroll B. Foster

TRANSFER PRICING WITH DIFFERENTIAL TAXATION

by

Carroll B. Foster*

The problems of transfer pricing are generally well known to most business school graduates and managers of vertically-integrated and multinational corporations, but may be a bit unfamiliar to some academic and forensic economists. The subject is typically relegated to later chapters of standard managerial economics textbooks, where microeconomic models of optimal transfer prices suggest that the problem is largely solved.

But the transfer-pricing problem is not solved. Increased globalization of production has not only made the subject more difficult, but has also generated more situations where forensic economists must come to grips with it in order to perform calculations necessary for certain tax, valuation, and litigation-related tasks.

The purpose of this paper is to introduce the reader to the subject of transfer prices, the reasons why they can present problems for litigation economists, and some general methods for dealing with those problems. Part I defines transfer prices, provides a brief historical overview, and explains the renewed interest in the subject in our era of multinational business. Part II reproduces two common theoretical models of optimal transfer prices and lists transfer-pricing policies commonly used in practice. The possibility of manipulating transfer prices to lower overall corporation tax liability is demonstrated. This manipulation is the source of most of the problems confronting forensic economists wrestling with transfer-pricing policies of clients and adversaries. Part III reviews IRS and OECD transfer price guidelines for multinational enterprises. These guidelines serve equally well for analysts and consultants striving to adjust recorded transfer price, cost, and profit data so as to measure values and damages in ways that are realistic, fair, and acceptable to the courts. Part IV summarizes the paper.

I. Introduction to Transfer Pricing

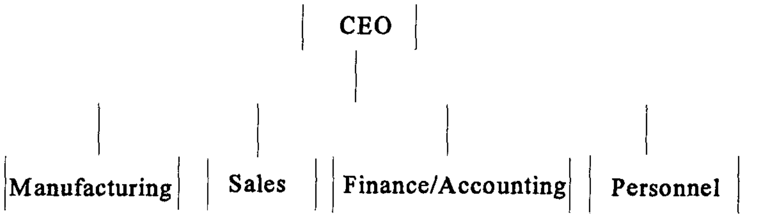
A. Corporate Structure

The Coasian view of a firm pictures a hierarchical organization within which authority flows from the top down. Internal contracting and market-type transactions are replaced by routine procedures and "bureaucratic" operating rules, yielding efficiencies from smoother information flows, avoidance of strategic behavior, and the possibly easier evaluation of the firm's managers and component parts.

By the late 1800s, a typical corporation had adopted a centralized unitary structure,

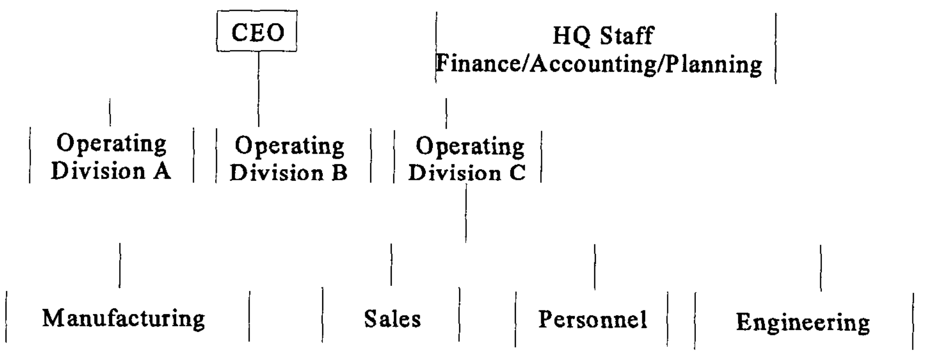
* The author is a lecturer in economics at the University of California, San Diego and would like to thank Matthew Lynde and Joseph D'Antoni for suggestions and comments which improved this paper in numerous ways, and Robert Trout for posing the question which stimulated this research in the first place.

or "U-form"[1] of organization, built around functional departments:



Although suitable for medium-sized single-product firms, the U-form became unwieldy as firms expanded into multiple products and markets. It was hard to measure efficiency or productivity in each department, and department managers might not cooperate to ensure maximum profit. Diseconomies of scale set in.

By the second decade of the 20th Century, larger multi-product corporations were experimenting with a decentralized multi-divisional structure, the "M-form," organized around semi-autonomous operating divisions:



Each operating division is a "profit-center" with sufficient local authority to maximize divisional profit, much like an independent single-product firm. Managers are closer to operations for which they are responsible, and evaluation of the company's component parts can now be based on recorded divisional profits. Corporate headquarters focuses on long-range planning and investment, handles administrative overhead costs of the divisions, and provides rules for how divisions should relate to one another.

B. Transfer Pricing

The M-form structure posed new management and control problems as firms

[1] The nature and history of U-form and M-form business structures are analyzed in Williamson (1975), ch. 8.

integrated forward from manufacturing to distribution or backward into resource extraction and materials supply. Now one operating division is supplying some or all of its output as an intermediate good input to another division. At what price or charge should this internal transfer of product take place? If the decision is left to negotiation by divisional managers and there is no external market for the product in question, a bilateral monopoly situation results, with the supplying division insisting on a high transfer price and the receiving division equally adamant in favoring a low one. Corporate headquarters can avoid the well-known conflicts and uncertainties of bilateral monopoly by dictating the transfer price, but this reduces divisional autonomy and may result in disgruntled divisional managers and a sacrifice of M-form efficiencies.

Typically, the umbrella corporation establishes a policy whereby transfer prices can be chosen and altered when necessary, perhaps by the divisional managers. Ideal transfer-pricing will be feasible and easy to implement, and will yield the following desirable results:

- Internal coordination -- the receiving division wants the same quantity of transfer product that the selling division wants to provide;
- Profit -- the agreed-upon quantity of transfer product is the amount that maximizes overall corporate profit; and
- Evaluation and fairness -- divisional managers, being evaluated for promotion and bonuses based on divisional costs and profits, do not see the transfer price as being unfair.

As one might suspect, ideal transfer-pricing policies are not easy to come by, and most M-form firms struggle to find merely satisfactory ones. Information requirements and disclosure problems are substantial, and conflict often arises between the profit and evaluative fairness *desiderata*. Richard Eccles, a student of the subject, writes that the "transfer pricing problem is a difficult and frustrating one...many managers regard it as unsolved or unsolvable."[2]

C. Differential Taxation

Increased globalization of corporate production added a new twist to the transfer pricing problem that made the subject of greater importance to forensic economists. Many modern transnational corporations are M-form with semi-autonomous subsidiaries located in different nations. Subsidiary profits are often subject to different tax laws, and this fact will influence the umbrella corporation's transfer prices and after-tax profit:

> Nearly all nations tax profits of subsidiary companies, but they do so at different rates. A corporation may use transfer-pricing policy as a means of concentrating profits in subsidiaries operating in low-tax areas. A subsidiary operating in a low-tax country will have its profits enhanced by charging a high price on transfers to subsidiaries in other countries and/or

[2] Eccles (1985), p. 1.

by paying a low price on transfers from other subsidiaries.[3]

Tax authorities do not always catch or challenge such tax-avoidance practices, especially if the practice raises the taxable base of subsidiaries in their jurisdiction. Furthermore, tax rate differentials may influence transfer-pricing even when transfers are not across international borders. Operating divisions in the U.S. are subject to state business taxes, which vary dramatically from one state to the next. And if the transfer product is a mineral or energy substance subject to *ad valorem* state severance taxes, there is again a clear incentive to fix a lower transfer price.[4]

If a corporation's transfer-pricing policy artificially raises or lowers subsidiary profit, use of subsidiary financial and accounting records by analysts becomes problematic. Business appraisers will have difficulty in using such data to arrive at a subsidiary's fair market "stand alone" value; forensic economists will be equally confounded in determining realistic revenue and profit losses in breach of contract and business interruption litigation.

D. History[5]

DuPont instituted a variant of the M-Form structure in 1921, and General Motors had done likewise by 1925. Such companies encountered the transfer-pricing problem from the outset. The accounting profession first explicitly addressed the special circumstances of M-Form structures, including transfer pricing, in 1925. Surveys summarized by Eccles suggest that perhaps 85 percent of large manufacturing firms confront the transfer-pricing problem today.

In nations where business profits were taxable, tax codes kept pace with business developments. The possibility of using transfer prices and other M-form management policies for tax avoidance was recognized in the U.S. as early as 1917. By 1921, the IRS could reallocate taxable income among affiliated companies and subsidiaries. The "Arm's Length Principle," the standard favored by the federal government for altering artificial transfer prices and profit distribution among subsidiaries, was adopted in 1935.[6] Initially, IRS transfer price regulations were applied mostly to domestic M-form corporations, but by the 1960s, the focus was changing to subsidiaries of multinational enterprises. Current

[3] Yunker (1982), p. 13.

[4] About 45 states have corporation profit taxes. Three states tax corporation gross receipts; Michigan uses a value-added "single-business tax." Severance taxes are important in Texas and Wyoming. States also differ in the formula used to calculate the state profit tax liability of multinational corporate subsidiaries doing business in the state. See Fisher (1996), ch. 17.

[5] This material is drawn from Eccles (1985), ch. 2, and *U.S Transfer Pricing Guide* (1995), §115.

[6] This principle is the basis for the transfer price guidelines for forensic economists discussed in Part III.

IRS transfer pricing regulations, in Code Section 482, were issued in 1968. Revisions of Section 482 in 1994 to incorporate the latest interpretation of the arm's length standard have also been adopted by the OECD.

II. Setting Transfer Prices -- Theory and Practice

Optimization methods used in microeconomic theory can be used to derive optimal (profit-maximizing) transfer prices for various internal and external market scenarios. This section presents models for two such scenarios. It then shows how strategic manipulation of the transfer price can increase after-tax corporate profit when the buying and selling divisions are subject to different corporate tax rates. A brief review of transfer-pricing rules commonly employed in actual business practice appears at the end of this section.

A. The Setting for Analysis

Assume that a vertically-integrated firm consists of two divisional profit centers, S and B. Selling division S supplies product X. Buying division B uses input X to produce final product Q, which is sold to the company's external customers. Corporate HQ generates no revenues of its own, and its own costs are independent of divisional operations.[7] It will be helpful to adopt the following notation:

X_s = total quantity of product X produced by S
X_t = quantity of X transferred from S to B
X_e = quantities of X sold or bought in external market
X_b = total quantity of X purchased by B
P_t = internal transfer price of X_t
P_e = external market price of X_e
$Q = f(X_b)$ = quantity of final output produced by B
$C(X)$ = total cost of producing X in S division
$R(Q)$ = total final sales revenue to B division
$N(Q)$ = net cost (excluding X) of producing Q in B division
t_s = corporate tax rate on S profits
t_b = corporate tax rate on B profits

The models below are first derived under the assumption of no taxation, then later modified to incorporate the effects of tax differentials.

B. Model 1 - No External Market for X

With no external market for X, $X_e = 0$ and $X_s = X_t = X_b = X$. In this rather typical

[7] HQ costs are set at zero in the two models which follow. These models represent opposite ends of a spectrum of possibilities regarding an external market in the transfer product. Models like these are standard fare in managerial economics textbooks. See Mansfield (1996), pp. 495-502, for examples and useful discussion of potential complications.

case, the selling and buying divisions are captive to one another in a bilateral monopoly setting where corporate HQ will have to resolve disagreements about the "optimal" equilibrium.

Selling division profit and the first-order condition for divisional profit maximization are as follows:

$$\pi^s(X) = P_tX - C(X)$$

$$\pi^{s\prime}(X) = P_t - C'(X) = 0 \Rightarrow P_t = MC_x$$

The manager of S division will want to supply a level of product X where the transfer price equals the marginal cost of producing X.

Buying division B's profit and the first-order condition for divisional profit maximization are as follows:

$$\pi^b(Q) = R(Q) - N(Q) - P_tX$$

$$\pi^{b\prime}(X) = [R'(Q) - N'(Q)]f'(X) - P_t = 0 \Rightarrow P_t = NMRP_x$$

The manager of B division will want to purchase a level of X where the transfer price equals the net marginal revenue product of X.

Corporation headquarters is assumed to set the transfer price to maximize total profit for the corporation as a whole. The relevant profit function and first-order condition are as follows:

$$\pi^{hq} = \pi^b + \pi^s = R(Q) - N(Q) - P_tX + P_tX - C(X)$$

$$\pi^{hq\prime}(X) = [R'(Q) - N'(Q)]f'(X) - C'(X) = 0$$

HQ wants a level of X which equates the marginal cost of producing X with the net marginal revenue product of X. By appropriate choice of P_t, this will be the level resulting from independent decisions by S and B, and the divisional conflicts are thus avoided.

Consider a simple numerical example with $C(X) = X^2$, $R(Q) = 140Q$, and $N(Q) = 12Q$, where $Q = 2\sqrt{X}$.

$$\pi^{hq} = 140Q - 12Q - X^2$$

$$\pi^{hq\prime}(X) = 128/\sqrt{X} - 2X = 0$$

Profit maximization results in $X = 16$ and $Q = 8$. HQ sets $P_t = \$32$. In division S, $MC_x = 32$ at $X = 16$, while $NMRP_x = 32$ at $X = 16$ in division B. Profits are $\pi^s = \$256$, $\pi^b = \$512$, $\pi^{hq} = \$768$.

C. Model 2 - Perfectly Competitive External Market for X

If there is a competitive external market for X with going price P_e, then S division can transfer product to B division or sell to the market, and B division can buy from S division or from the market: $X_s = X_t + X_e^s$ and $X_b = X_t + X_e^b$. With divisional autonomy, the only transfer-pricing policy is to set the transfer price at the market price ($P_t = P_e$), since S division will not voluntarily transfer X at less than P_e, while B division will not voluntarily accept transfers at more than P_e.

Using the Model 1 numerical example, but with $P_e = \$26$:

$$\pi^s(X_s) = 26X_s - X_s^2$$

$$\pi^b(X_b) = 140Q - 12Q - 26X_b,\ Q = 2\sqrt{X_b}$$

Independent choices by S and B result in approximate solution values of $X_s = 13$, $X_b = 24.2$, and $Q = 9.8$. B buys 13 units of X from S and 11.2 units in the market. $\pi^s = \$169$, $\pi^b = \$630$, and $\pi^{hq} = \$799$.

D. Strategic Transfer Pricing Induced by Differential Tax Rates

There are possibilities for strategic manipulation of P_t in the presence of differential corporate profit tax rates. Suppose divisions S and B are in separate tax jurisdictions with rates t_s and t_b, respectively. If corporate headquarters is willing to reduce divisional autonomy, transfer prices can be set to shift profit artificially from one division to the other in order to reduce overall corporate tax liability. This possibility can be illustrated by the numerical examples for the two models above, with $t_s = 10\%$ and $t_b = 20\%$.

In Model 1, with X = 16 and $P_t = \$32$, after-tax divisional profits were $(1-t_s)\pi^s = \$230$ and $(1-t_b)\pi^b = \$410$, with after-tax total profit of \$640. HQ could mandate that X = 16, then raise P_t to shift profit from B to S. At the extreme, where $P_t = \$64$, $\pi^b = \$0$, and total corporate net profit rises from \$640 to $(1-t_s)\pi^s = \$691$.

This policy deprives the S and B managers of some of their previous autonomy. Nevertheless, if both are evaluated in terms of divisional profit at a (possibly unpublished) transfer price of \$32, management incentives for cost efficiency can be retained, while the company as a whole will enjoy a substantial gain in after-tax profit.

In Model 2, original divisional net profits were $(1-t_s)\pi^s = \$152$ and $(1-t_b)\pi^b = \$504$, with after-tax total profit of \$656. Suppose that HQ mandates that S produce 13 units of X for transfer to B and that B continue to produce Q = 9.8 while accepting 13 units of X from S at a fixed transfer price P_t. At the extreme, where $P_t = \$74.47$, $\pi^b = \$0$ and total net profit rises from \$656 to $(1-t_s)\pi^s = \$719$.

In the situations just depicted, it was possible for the corporation to find a profitable set of quantity decisions, then rig the transfer price to further reduce tax exposure. Doing so encroaches on some of the efficiency-inducing autonomy of divisional managers, at least one of whom will view the resulting transfer-pricing policy as manifestly unfair! The extent to which artificial pricing takes place depends on internal corporate tradeoffs and on the astuteness of the relevant tax authorities. In the cases illustrated, the tax administration with jurisdiction over the S division, whose profits are inflated, would not challenge the practice. By the same token, the accounting records of the B division would be highly misleading to an appraiser or forensic economist trying to use them.

E. Transfer Pricing in Practice

Except in cases where a competitive external market sets the transfer price, theoretical models like those above are of little practical use to managers. Firms do not have the luxury of continuously differentiable functions and seldom have accurate knowledge of the relevant marginal quantities. Nor have mathematical programming methods and other "academic" approaches been of much help. Most companies make do with suboptimal -- but simple -- rules of thumb to arrive at usable transfer prices.

In summarizing corporate practices, Eccles divides transfer-pricing policies into cost-based and market-based methods. Cost-based methods build the transfer price from supplying division variable or full unit cost. The particular formula may be based on actual cost or standard cost, and may or may not add a fixed markup to match the company's current or target return on investment. Market-based methods start from an external price of transactions comparable to the internal transfers, perhaps discounted for estimated selling expenses not incurred on internal transactions.[8] Penelope Yunker's survey of multinational corporation transfer-pricing policies found that market-based methods were by far the most widely used, followed by markups on standard full and actual full unit cost. Allowing divisional managers to negotiate the transfer price between them is also common.[9]

Of course, consulting economists encountering transfer pricing in litigation-related contexts need not determine if a firm's transfer-pricing policy is optimal. For them, the problem is whether recorded transfer prices reflect economic realities in ways acceptable to the courts and taxing authorities, or are accounting artifices to reduce over-all corporate tax exposure. In this vein, it is disturbing to note that Eccles found a number of instances of "dual pricing" policies where, typically, average total or variable cost was charged to the buying division and a market-based price was credited to the selling division.[10] As seen above, some sort of dual transfer pricing is exactly what would be indicated by Models 1 and 2 when the HQ corporation wants to reduce tax exposure without penalizing one or more profit center managers during the evaluation process.

Thankfully, the practice of artificially manipulating transfer prices as part of tax-avoidance or other corporate strategies may be relatively rare. Yunker writes:

> Multinational companies are generally conscious of the potential

[8] Eccles (1985), ch. 2 and Table 2-2. Standard costs are projections of what actual costs would be if a firm or division operates at some defined "efficient capacity."

[9] Yunker (1982), Table IV.C.2. Use of negotiation, given the problems of bilateral monopoly, is practically a decision to have no transfer price policy at all. The finding that perhaps 42 percent of M-form firms use full (average total) cost as a base for transfer prices is perplexing, since one useful contribution of economic theory to this subject is to show that this policy is <u>never</u> optimal. [See Eccles (1985), pp. 40-42, and Tang (1979), ch. 2, 5.] An explanation could be that full-cost pricing is both simple and allowable by U.S. and OECD tax authorities.

[10] Eccles (1985), pp. 8-9, 77.

> opportunities that exist for the enhancement of corporate goals through active manipulation of transfer prices, but they are also strongly cognizant of the obstacles to successful manipulation in the legal and regulatory environment.... There is certainly some utilization of transfer prices for instrumental purposes in the real world, but the indications are that this is not considered one of the principal instruments of business policy by most corporations.[11]

III. Tax Guidelines for Adjusting Transfer Price Data

The essential problem of transfer prices for forensic economists is the possibility that they reflect corporate tax reduction or other strategies rather than underlying economic realities and true opportunity costs. This issue, which goes beyond routine product transfers from one division to another to include one-time transfers of tangible and intangible property, is clearly recognized by taxation authorities:

> Transfer pricing is the term used to describe the prices that related parties set for goods, services, loans, intangibles, and property rentals when engaged in transactions among themselves. Because these prices are not negotiated in a free, open market, it is possible that they may deviate from prices agreed upon by non-related parties in comparable transactions under the same or similar circumstances, In addition, multinational enterprises sometimes attempt to use transfer prices to subject as much profit as possible to tax in low-tax jurisdictions.[12]

A. The Arm's Length Principle

Tax administrations of many nations have given themselves authority to reallocate taxable profit among profit centers and subsidiary corporations in instances where initially reported profits appear to be the result of artificial internal pricing and property transfer policies. Developed nations, including the United States and those of the European Common Market, have also reached wide agreement on a standard, the Arm's Length Principle, to be used for such profit reallocation:

> Where business is transacted between two persons, one of whom controls the other or both of whom are controlled by a third person, the arm's length principle requires that the prices and terms of the transaction

[11] Yunker (1982), p. 40. Note also that Eccles (1985), in a small survey, found infrequent use of dual-pricing policies.

[12] *U.S. Transfer Pricing Guide* (1995), §100. Unless otherwise noted, all material on tax guidelines herein is taken from this Commerce Clearing House publication, which explains Internal Revenue Code Section 482 and provides sample calculations for many of the adjustment methods discussed below.

> should not differ from the prices and terms which would prevail in transactions between unrelated parties. An arm's length price is simply the price independent parties would charge each other under the same or similar circumstances.[13]

Similar wording is found in Article 9 of the OECD Model Tax Convention. OECD tax guidelines go on to say that "the arm's length principle follows the approach of treating the members of [multinational enterprises] as separate entities rather than as inseparable parts of a single unified business."[14] This points to the common alternative to the arm's length standard, called "formulary apportionment":

> Despite international consensus on the applicability of the arm's length approach to transfer pricing, some dissenters, most notably certain states within the United States, have chosen to approach the issue of transfer pricing through formulary apportionment, also known as unitary taxation. Under formulary apportionment, the profits of various branches of an enterprise or the various corporations of a group are not calculated as if the branches or subsidiaries were distinct and separate entities dealing at arm's length with each other, but rather the entire group is regarded as a unity.[15]

Arm's length has been the standard of choice by the IRS since 1935 (and is now also the rule in Europe) whenever transfer-pricing policy necessitates adjustments to corporate income and financial records. Unless a specific local court dictates otherwise, the arm's length standard should equally serve as the guiding principle for forensic economists:

- Federal and international acceptance of the arm's length standard suggests that any profit adjustments in accordance with its principles should be acceptable (if not mandatory) in most courts;
- The philosophy of the arm's length standard is that of efficient market dealings between independent parties, and adjustments to profit records based on the standard are more likely to reflect the true economic positions and opportunity costs of corporate subsidiary plaintiffs and defendants than formulaic adjustments based on local unitary tax regulations;
- The formulas used by unitary tax jurisdictions differ from one another in somewhat arbitrary ways and enjoy no domestic or international consensus.
- Litigation economists and business appraisers, who are accustomed to revising business accounting records in the normal course of their work, will undoubtedly be familiar with the types of revisions approved by tax authorities in conjunction

[13] *U.S. Transfer Pricing Guide* (1995), §105.

[14] OECD (1995), p. I-3.

[15] *U.S. Transfer Pricing Guide* (1995), §110.

with the arm's length standard.

B. Arm's Length Adjustment Methods

IRS and OECD regulations list a number of transfer pricing methods which can be used by firms preparing tax records in accordance with the arm's length standard, but do not mandate any particular one. Section 482 establishes the Best Method Rule which, "at its heart, merely requires the use of the pricing method which provides the most reliable measure of the arm's length result."[16] In practice, two or more of the methods should be used to cross-check reasonableness and accuracy.

Tax codes recognize five categories of internal transfer activities:

1. Transfers of tangible property;
2. Transfers of intangible property;
3. Provision of services for related parties;
4. Intercompany loans or discounts;
5. Intercompany property rentals.

Allowable transfer pricing methods for property transfers are divided into transactions-based and profit-based methods. Transactions-based methods consist of comparable uncontrolled price (CUP), comparable uncontrolled transaction (CUT), resale price method (RPM), and cost-plus. Profit-based methods include comparable profit method (CPM) and profit-split.[17]

The economist or analyst who is engaged in revising a client corporation's financial data and transfer pricing records will clearly be concerned with what is or is not a comparable external (uncontrolled) transaction. The tax guidelines list five categories of factors to be considered in judging the comparability of an external transaction to an internal transfer:

1. Property or service -- the nature of the transferred commodity. An issue is whether certain intangibles are embedded in tangible property being transferred.
2. Function -- the economically significant activities of parties pursuant to the transaction, including R&D, transportation, and administrative functions.
3. Risk -- the level of market, foreign exchange, credit and liability risk exposures of

[16] *U.S. Transfer Pricing Guide* (1995), §240.

[17] CUP, RPM, cost-plus, and profit-based methods are appropriate for tangible property transfers; CUT and profit-based methods for intangible property transfers. With suitable modification, these methods can be applied to internal transactions involving provision of services, loans, and property rentals. CUP and CUT are considered the most reliable when applicable. See *U.S. Transfer Pricing Guide* (1995), §235. Paulsen (1989), pp. 112-13, discusses a Financial Model Method (FMM) which is usable for determining arm's length patent royalty rates and intangible property transfer prices.

parties to the transaction.

4. Contract terms -- credit, warranty and payment clauses governing the transaction.
5. Economic conditions -- geographic market factors and business cycle conditions which influence transaction price or cost.

Comparable uncontrolled price (for tangible property transfers), and comparable uncontrolled transaction (for intangible transfers) are, when applicable, the preferred transfer price adjustment methods. Both are predicated on the existence of an external market for the transfer product, with available transactions data either between unrelated agents in that market or between the market and one of the subsidiary profit centers. IRS comparability standards are higher for CUP and CUT than for any of the other arm's length transfer-pricing methods listed.

For transfers of tangible property, either the resale price or cost-plus method may be used to reallocate subsidiary profits when sufficiently comparable data for external transactions in the transfer product itself are not obtainable. Consider our buying division B and selling division S from Part II above, where B sells or distributes to the market at large. Assume that corporate transfer-pricing policy makes division B and S profits unrealistically high or low. RPM adjusts taxable B profit to the point where B's gross profit margin is approximately equal to that of similar unrelated firms, and adjusts taxable S profit commensurately. If it is easier to find firms comparable to S than to B, the cost-plus method builds a transfer price based on S division unit costs with an arm's length markup added, then recomputes taxable B and S profit accordingly.

Profit-based methods may be something of a last resort:

> As transfer pricing has grown in prominence, the need for profit-based methods has become more and more apparent. It is readily agreed by almost all practitioners that transaction-based methods are preferable because they are more likely to produce arm's length results. However, a similar consensus agrees that, for many transactions, few or no suitable comparables are available to serve as the basis for applying a transaction-based method.[18]

The comparable profit method adjusts subsidiary profits to correspond to the profit rates of similar uncontrolled companies in the external market. CPM attempts to achieve an arm's length equivalent operating profit, as measured by EBIT/Sales or return on assets.[19] The method is highly criticized because of the difficulty of assessing the operating efficiencies of firms being compared, but the focus on financial ratios and rates of return will be familiar to most economists.

[18] *U.S. Transfer Pricing Guide* (1995), §500. A lack of suitable comparables may arise with unique products, or because the external market is itself dominated by M-form vertically-integrated multi-national companies.

[19] Note the contrast with RPM, which is couched in terms of gross profit.

The profit split method is by far the least favored of the adjustment methods allowable in transfer pricing cases. The *U.S. Transfer Pricing Guide* reports that the "IRS has been loath to approve use" of the method, and that:

> ...the profit split method is highly likely to produce a result that is not arm's length. Its primary selling point seems to be its simplicity as well as the fact that application ... relies entirely on data internal to [the firm], rather than on comparable transactions.[20]

The tax guidelines mention two profit split models. The seldom useful "comparable split" approach finds two independent companies that stand in the same relationship to one another as the B and S divisions in the controlled firm. Relative profit shares between the independent firms are used as a guide to reallocating profit between S and B. The more common "residual split" first estimates profits the S and B divisions would likely earn based on their functional activities (manufacturing, transportation, marketing, and so on). This amount is subtracted from total corporation profit, and the residual is then split between S and B based on their relative estimated contributions of intangible assets to the firm's operation.

IV. Summary and Conclusions

Many large corporations, including perhaps most multinational enterprises, have chosen an M-form structure in which the company is divided into two or more semi-autonomous divisions or subsidiaries called profit centers. When one profit center transfers product or property to another, a transfer price is charged to one as a cost and credited to the other as revenue. If the profit centers are located in different tax jurisdictions, the company policy may be to manipulate transfer prices so as to report a lower taxable base by the subsidiary in the higher tax area. Such strategic manipulation of transfer prices generally produces the result that the accounting profits and financial records of the individual subsidiaries do not reflect their true opportunity costs, income potential, or fair market values.

With increasing frequency, practitioners in the field of litigation economics are finding themselves in the position of working with divisional profit centers as if they were stand-alone companies.[21] This can be problematic if a substantial proportion of the division's costs or revenues are dictated by an umbrella corporation's transfer-pricing policy and the transfers routinely cross international boundaries or the boundaries of different domestic tax jurisdictions. If the transfer prices have been set to reduce over-all corporate

[20] *U.S Transfer Pricing Guide* (1995), §525. Actually, the comparable split method does require data from external sources.

[21] Lynde (1996) details a number of instances where this situation might arise in cases involving patent damages, including the possibility that "a wholly-owned foreign subsidiary may be the sole plaintiff in that foreign country against either the infringer's wholly-owned subsidiary or against the subsidiary and the parent combined."

tax liability, then the records of the profit center may require extensive adjustment before good estimates of sale values or economic damages can be calculated.

The forensic economist might deal with transfer pricing problems by first answering a series of preliminary questions:

- First, does the business in question engage in product, property or service transfers with another subsidiary or profit center under the same umbrella or headquarters corporation?

If the answer is yes, then:

- Do the transfers cross from one relevant tax jurisdiction to another?
- What transfer-pricing policy is currently in use? In particular, does the company use dual pricing?
- Has the transfer pricing policy in use been challenged by tax authorities in the subsidiary's jurisdiction or in the jurisdiction where the courts will try the case?

At least two problems may surface. One is that the transfer prices have been set with tax avoidance in mind. The other is that the transfer-pricing policy is in complete accordance with tax regulations, but is based on some full cost formulation. In either case, the accounting data of the company may need substantial revision if an accurate picture of the company's true economic prospects is to be derived. In both cases, the analyst will want to recompute costs, revenues and profits using a transfer-pricing scheme which is not only permitted by the tax authorities and/or courts, but which is likely to produce economically meaningful and defensible results.

If the relevant jurisdiction is in the United States or any nation of the European Economic Community, then the allowable methods of revision of transfer prices are listed and explained in the *U.S. Transfer Pricing Guide* and the OECD's *Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations*. Any revisions undertaken by the forensic economist should satisfy the "arm's length" principle. There seems to be both a theoretical and an administrative preference for using the comparable uncontrolled price or transaction (CUP or CUT) methods where possible. Second choice would be the resale price or cost-plus methods. With the latter, something other than full costs should be used; average variable costs are usually closer to the marginal costs suggested by economic theory. Profit splitting methods, whereby the economist simply recomputes what the subsidiary's profits should have been but for artificial transfer pricing, are to be avoided except as a last resort. They are deceptively simple to apply, but the results are essentially arbitrary. Splitting methods are often challenged in tax courts, and might not hold up well in civil courts.

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VALUATION OF WARRANTS WITH IMPLICATIONS TO THE VALUATION OF EMPLOYEE STOCK OPTIONS

by

Gary R. Johnston*

I. Introduction

This study examines the valuation of stock options and warrants. It discusses those factors relevant to the value of call options, and it explains and discusses the most widely recognized model in use for the valuation of stock options. The predictive ability of the Black-Scholes and Shelton option models are compared. Also discussed is the volatility measure used in the Black-Scholes model since business appraisers find this measure the most difficult aspect of using the Black-Scholes model.

So far business appraisers have not been able to find a stock option-pricing model reliable enough to value employee stock options on privately held stock. The Financial Accounting Standards Board in Accounting for Stock-Based Compensation, Statement of Financial Accounting Standards No. 123 (1995) (FASB 123), recommends the minimum value method to predict employee stock option prices when the underlying stock is privately held. Minimum value can be determined by a present value calculation. It is the current price of the stock (S_0), reduced by the present value of the expected dividends on the stock, ($e^{-\delta t}$), if any, during the option's term, minus the present value of the exercise price (Xe^{-rt}). Present values are based on the risk-free rate of return. The minimum value option price is:

$$C_m = S0e^{-\delta t} - Xe^{-rt}$$

Where:

C_m = minimum value of option,
S_0 = current stock price,
δ = annual dividend yield,
t = option term to expiration,
r = risk-free interest rate,
X = exercise price.

The minimum value also can be computed using an option-pricing model and an expected volatility of effectively zero. (Standard option-pricing models do not work if the volatility variable is zero because the models use volatility as a divisor, and zero cannot be a divisor). Because the minimum value method ignores the effect of expected volatility, it differs from methods designed to predict option prices, such as the Black-Scholes and binomial option-pricing models and extensions or modifications of those original models. This study is the first research that supports the minimum value method for pricing employee stock options on privately held stock. The empirical data suggests that the FASB 123 minimum value method will result in reasonable estimates of employee stock option

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prices when the stock option is in-the-money. In addition, this study will show that both a linear model and a constant volatility input measure to the Black-Scholes model will result in reasonable estimates of in-the-money employee stock option prices.

For employee stock options on publicly traded stock, FASB 123 recommends that historical volatility be calculated over the most recent period that is generally commensurate with the expected option life. This calculated expected volatility measure is then input into the standard option-pricing models. Internal Revenue Procedure 98-34 sets forth the methodology for valuing employee stock options for estate, gift, and generation-skipping transfer tax purposes of non-publicly traded employee stock options on stock that is publicly traded. Taxpayers may use option-pricing models that employ factors similar to those established by the Financial Accounting Standards Board in Accounting for Stock-Based Compensation, Statement of Financial Accounting Standards No. 123 (1995). This study will show that the implied volatility estimate is relatively low for both in-the-money and out-of-the-money long-term options. Accordingly, the use of historical volatility measures of the underlying stock or stock of similar companies will not necessarily lead to reasonable stock option price estimation.

Many courts are finding option-pricing models, including the Black-Scholes model, difficult to apply. In *Lewis v. Vogelstein*, Court of Chancery of Delaware, 699A. 2d 327; 1997 Del., the court would not apply the Black-Scholes model to value non-publicly traded restricted options. The family law case of *Chammah v. Chammah*, in the Superior Court of Connecticut, Stamford, FA95145944S, involved the valuation of employment stock options. Again, the court did not allow the use of the Black-Scholes model to value the options. Similar results occurred in *Wendt v. Wendt*, Superior Court of Connecticut, Stamford, 1997 Conn. FA960149562S. In the case of *Spicer, Brittain, and Buys-MacGregor v. Chicago Board Options Exchange, Inc.*, No. 88 C2139, United States District Court for the Northern District of Illinois, 1990 U.S. Dist., the court could not find the proper volatility input measure to apply the Black-Scholes model.

II. Background

Warrants, like employee stock options, provide the holder the right to buy the stock at a stated price. Employee stock options are generally issued by a company as part of an incentive compensation program or in conjunction with raising capital. Valuing employee stock options may be necessary in the following situations:

- When they are granted, exchanged or terminated.
- For disclosure purposes including determination of compensation expense under FASB 123.
- Divorce of the option holder.
- Gift of the option to a trust.
- Determination of compensation for SEC or income tax purposes.
- Repurchase of the option by the company.
- Litigation damages resulting from breach of employment contracts in which the option value is at issue.

Business appraisers have used warrant pricing models to value employee stock options issued by privately held companies. However, employee stock options do differ from warrants in several ways:

- There is no public trading market for the option and most are non-transferable.
- They generally have more than a year remaining until expiration.
- They may be forfeited on employee termination.

Employee stock options provide the holder the right to buy a stated number of shares of stock at a stated price within a predetermined time period. Generally, employee stock options cannot be sold to another investor; the holder must exercise the option to realize its value through the subsequent sale of the underlying stock. The exercise price is the stated price for the option holder. Employee Stock options generally have the following characteristics:

- They are options to purchase common stock of either a publicly held or privately held company, but there is no public trading market for the option. Accordingly, a marketability discount may be required.
- They are generally issued with three to ten years remaining until expiration.
- They are issued with vesting, forfeiture, and inalienability restrictions.

Intrinsic value is the difference between the current underlying stock price and the exercise price of the option but is never less than zero. If the value of the underlying stock is above the exercise price, the option is referred to as being in-the-money. When the stock price and the exercise price are equal, the option is referred to as being at-the-money. If the value of the underlying stock is less than the exercise price, the option is referred to as being out-of-the-money. For at or out-of-the-money options, the intrinsic value is zero but the option may still have time-value. The adjusted intrinsic value of an option is the intrinsic value of the option prior to expiration. The adjustment takes into consideration the time-value of money, since the exercise price will not be paid until the expiration date. The adjusted intrinsic value is:

$$S_0 - Xe^{-rt}$$

Where:

S_0 = the current underlying stock price,
X = the exercise price,
r = the continuously compounded annual risk-free interest rate,
t = time in years to option expiration.

Time-value is the value of the option above the intrinsic value at any time before the option expiration date. Most of the time-value of an option comes from the probability that the option will finish in-the-money. Even if the option is out-of-the-money, there is still a chance that the stock price will rise and the option will have intrinsic value at the expiration date. The time-value of the option is the greatest when the option is at-the-money (the stock price and the exercise price are equal).

Volatility is a significant parameter in option valuation models. As volatility increases the probability that the stock price will exceed the exercise price increases. The volatility parameter is generally estimated based upon historical variability of stock returns. Historical prices, however, are not generally available for privately held companies. Using the volatility measures of comparable public companies or companies within the same industry may not be acceptable substitutes. The difficulty in estimating the volatility measure for privately held stocks has limited appraisers' use of the Black-Scholes model. Many business appraisers, therefore, advocate the use of the Shelton model.

No replacement for the binomial or Black-Scholes option-pricing models has been generally accepted by the valuation community. The Financial Accounting Standards Board and the Securities and Exchange Commission have endorsed the Black-Scholes option-pricing model for determining option value. Originally, the model was developed to price marketable short-term European style options on non-dividend paying securities.

Fischer Black was one of the early researchers of option-pricing theory. He first applied the Capital Asset Pricing Model (CAPM) to the valuation of warrants and the underlying stock. He used a differential equation to compare the rates of change between the underlying stock and the warrant under the assumption that both were priced according to the CAPM (Black 1989). He later teamed up with Scholes (1973), and their combined research indicated that neither risk nor expected return belonged in the equation, since risk and return canceled each other out. For example, two stocks each selling for $100 today have been priced by investors after they have considered each stock's risk and its expected future prices. Higher risk cancels out higher expected return and leads to the same current price for a high-risk stock and a low-risk stock. Black and Scholes concluded that the expected gain on a stock option or warrant did not matter in determining what the current price should be for the stock option or warrant. This insight allowed them to solve the option valuation equation.

The Black-Scholes option-pricing model (1973) is based upon the following assumptions:

1. Asset prices adjust to prevent arbitrage.
2. Stock prices change continuously.
3. Stock prices follow a lognormal distribution.
4. Stock returns follow a standardized normal distribution.
5. The model is used for European style options on non-dividend paying stocks. These options have no early exercise privileges.
6. Interest rates and volatility of stock returns remain constant over the option life.
7. Investors evaluate their investment gains in terms of percentage returns.
8. The return generating process is an unbounded random walk with a trend, where the trend is the expected growth rate of the stock return.

The Black-Scholes model assumes that common stock warrants are similar to European options which cannot be exercised prior to the expiration date. However, warrant investors do exercise early if the underlying stock pays a large enough dividend. Although there are many problem areas in the pricing of warrants (e.g. call provisions, expiration extensions), use of the Black-Scholes model does require an estimate of the appropriate risk-free rate of return and the underlying stock return volatility.

III. Literature Review

Louis Bachelier's doctoral dissertation in 1900 is the earliest analytical option-pricing approach. Bachelier assumed that the stock price behavior process followed a Brownian motion process and that stock returns had a normal distribution. Although his model failed to account for the time value of money, he laid the foundation for the development of future option-pricing models (Bernstein 1992).

Sprenkle's (1961) work built on Bachelier's foundation by assuming that stock prices are lognormally distributed and allowing for drift in a random walk. Boness (1964) extended Sprenkle's work, by considering the time value of money, whereby he discounted the expected terminal stock price to present value using an expected rate of return on the stock. Later, Samuelson (1965) extended Boness's model by allowing the option to have risk levels different from that of the stock.

Volatility has a relationship with the expected value of the stock. Merton (1973) showed that the more risky stock has a more valuable warrant. Sprenkle (1961) and Kassouf (1965) show that the value of the warrant increases with greater stock price variability. Van Horne (1969) recognized the positive effect of higher interest rates on warrant value. Research does not support the contention that stock price volatility and warrant price changes are positively related. Van Horne (1969), and Melicher and Rush (1974) found a significant positive relationship, but Shelton (1967) did not.

In 1973 Merton resolved the dividend issue for European option-pricing models and also considered variable interest rates. Cox and Ross (1976) argued that stock prices don't move as a diffusion process whereby price changes are continuous from one point to another, but prices can jump. Their idea was subsequently expanded into the Cox, Ross, and Rubinstein (1979) binomial model. The jump process for stock movements (Cox and Ross 1976) is an extension of the Black-Scholes model whereby the lognormal stock price process is a special case of the jump process.

Merton (1976) combined the jump process and a diffusion process after each jump. He constrained the jump process to form a modified Black-Scholes model by assuming that the sizes of the jumps are distributed lognormally. In 1987, Hull and White, and Scott and Wiggins developed generalized Black-Scholes models that allowed volatility to be a stochastic process.

There are only a few empirical studies on warrant pricing. Noreen and Wolfson (1981) used 52 observations of warrant prices to test the Black-Scholes model adjusted for stock dilution upon the future exercise of the warrants. Their primary focus was to price employee stock options through the use of warrant pricing models. Schwartz (1977) used a finite difference approach to a partial differential equation for pricing contingent claims based on warrant prices. Lauterback and Schultz (1990) tested both the Black-Scholes model with a dilution adjustment and a constant elasticity of variance model, and found the latter model superior. In 1997, Hauser and Lauterback found the constant elasticity of variance model superior in option price prediction, but also found the dilution adjusted Black-Scholes model a reasonable economical alternative.

IV. Analytical Approximation Models

Analytic approximation models involve estimating the premium for early exercise.

Analytical models for an American option model were developed by Roll (1977), Geske (1979), and Whaley (1982). The analytical solution prices the early exercise right provided in American style options. Other analytical models to value the option to exchange one asset for another soon followed with Margrabe's (1978) and Stuly's (1982) option valuation models on the minimum or the maximum of two risky assets. MacMillan (1986) suggested using a quadratic approach to valuing the early exercise right. Baron-Adesi and Whaley (1987) implemented the suggestion.

V. Shelton Model

$$W = [\text{Maximum Value} - \text{Minimum Value}][\text{Zone location factor}] + [Max(S\text{-}X,\ 0)].$$
$$W = [.75S - (MAX(S\text{-}X,0))]\ [\sqrt[4]{(t/72)}\,(\ 47\text{-}4\ 25\ D/S + .17(L))] + [MAX\ (S\text{-}X,0)]$$

Shelton (1967) developed a warrant-pricing model that does not require the use of a volatility measure to predict warrant price. Shelton's first steps were to establish a zone of plausible warrant prices based upon the relationship between the warrant price and its associated stock. The lower range of the price zone of a warrant was the price of the common stock minus the warrant exercise price but not less than zero, so the minimum price equals the maximum of:

$$W = Maximum\ [S\text{-}X,\ 0],$$

Where:

W = *minimum warrant price,*
S = *underlying stock price,*
X = *exercise price of warrant*

To establish the maximum price of a warrant, Shelton relies on the following assumptions:

1. A warrant price will equal its exercise value if the stock sells for four times its warrant price or more; and,
2. A warrant seldom trades above its exercise value.

Shelton then sets the warrant price upper limit at 75 percent of the underlying stock price or $0.75S$. The range of possible warrant prices is summarized as follows:

$$(S - X) < W < 0.75S, \qquad \text{if } S < 4X, \text{ or}$$
$$W = (S - X), \qquad \text{if } S \geq 4X.$$

Shelton used a multiple regression approach on a sample size of 99 warrants to develop his empirically based warrant-pricing model. Through trial and error he selected variables that predicted warrant prices located within the upper and lower limit of his plausible price zone.

The variables considered were time to expiration, represented by t, where t is stated in months; annual dividend yield, represented by D/S; warrant listing on an active exchange,

$L = 1$ if listed or 0 if not listed, a regression coefficient of 0.47; the stock price, S; and the exercise price, X.

To estimate the price of a warrant Shelton began by placing the warrant at 47 percent of the distance from the bottom of the zone associated with the price of the underlying common stock. He then reduced the location by the adjusted dividend yield and increased the location by the listing factor. Finally, he multiplied this adjusted location by the longevity factor.

Shelton's conclusions about the way warrant prices are determined for stocks selling below four times the exercise value were:

1. Virtually all warrant prices will fall within the plausible price zone.
2. Warrant prices may be located anywhere within this zone, but will, on average, be near the middle.
3. Variations within the zone cannot be explained by using linear regression techniques.
4. Dividend yield foregone by not owning the stock is the most significant factor in explaining location within the zone.
5. Shelton's methodology is considered most applicable to warrants with a life of more than one year.

VI. Black-Scholes Option-Pricing Model

$$C_E = SN(d_1) - Xe^{-r(t)}N(d_2),$$

Where:

C_E = *European call option price,*

$N(\bullet)$ = *cumulative normal distribution function,*

$d_1 = [\ln (S/X) + (r + .5\sigma_2)(t)] / \sigma\sqrt{t}$

$d_2 = d_1 - \sigma\sqrt{t}$

Therefore, the value of a call option must equal or exceed the stock price minus the present value of the exercise price:

$$C \geq S - Xe^{-r(t)}$$

From this general model, the Black-Scholes model adjusts the two components of stock price and the exercise price for risk. These adjusting probabilities, $N(d_1)$ and $N(d_2)$, modify the model to account for the uncertainty in the future stock price.

For deep in-the-money call options, d_1 and d_2 become large, and $N(d_1)$ *and* $N(d_2)$ approach 1. $N(d_1)$ also represents the option's delta. (Delta (Δ) is the change in the price of an option with respect to a change in the price of the underlying security). The continuous time version of the call option delta (Δ_c) is given by $\Delta_c = \delta c / \delta s = N(d_1)$. Also, delta can be approximated by the change in the price of the call option divided by the change in the price of the underlying stock over a short time interval such that $\Delta_c = (C_t - C_0)/(S_t - S_0) = \Delta C/\Delta S$,

Where:

C_t = *call price at time (t),*
C_0 = *call price at time (o),*
S_t = *underlying stock price at time (t),*
S_0 = *underlying stock price at time (0),*
ΔC = *change in call price over the time interval,*
ΔS = *change in stock price over the time interval.*

The delta of a call option ranges between *0* and *+1*. The option delta measures the slope of the option price function line. Delta is *0* when the call is deep out-of-the-money and is *+1* when the option is deep in-the-money. Delta becomes less sensitive to changes in the underlying stock price as the time to maturity increases. The option price function line tends to be approximately linear with longer option maturities. The delta measures the price sensitivity of the option to the underlying security price. If one variable in the option delta equation, d_1, is allowed to fluctuate while the other variables are held constant, it can be observed that the option delta is influenced by the following variables ranked in order of significance:

- *stock price (ln(S/X)),*
- *standard deviation (σ),*
- *time to expiration (t),*
- *expected return rate (r).*

Also the following observations can be stated for in-the-money warrants:

- Warrant prices increase at a decreasing rate as time to expiration increases and volatility increases.
- Warrant prices increase at an increasing rate as the exercise prices decreases.
- Warrant prices increase at a decreasing rate as time to expiration increases and exercise price decreases.

The Black-Scholes model has five input variables consisting of stock price, *S*, exercise price, *X*, time to expiration measured in years, *t*, annualized continuous risk-free interest rate, *r*, and the annualized standard deviation of the stock returns, σ. Any one variable can be determined when the other four variables are known. The implied volatility of the warrant represents the solution to the Black-Scholes model when the other variables and the option price are known.

Estimation of two parameters, the risk-free interest rate and the standard deviation, represent the greatest difficulty in using the model. In the long-term, the trend is the dominant determinant of the Brownian motion with drift process, while, in the short-term, volatility of the process dominates. Option value depends on the average size of the underlying asset price movement, volatility, not on the direction of the stock price movements. Thus, investors price the option equally provided there is agreement on the anticipated price change size.

The Black-Scholes option model requires an estimate of the underlying stock's future volatility over the term to expiration of the option. Since this parameter is unobservable, an estimate is generally made using historical or implied volatilities. The model, however, does not specify the historical period over which the volatility measure is calculated. Different historical selection periods will result in different volatility measures which may not represent investor expected volatility estimates. The model assumes that volatility is constant over the option life. But, it is possible for volatility to change over the option life.

Assuming investor's expectations of future volatility are based upon the recent past, more weight should generally be given to recent history than to earlier history since return volatilities do change. A long-term based average may generally be desirable assuming many volatility changes are temporary.

The Black-Scholes model assumes that the return generating process is stationary. But the jumps that stock prices experience on occasion may indicate that the return generating process is non-stationary not withstanding the Stable-Paretian hypothesis. A jump in stock prices may be considered to be similar to temporarily higher volatility. Black (1975) observed that volatility rises as stock price falls and volatility declines as stock prices increase. For long-term options or warrants, the trend of the stock price, not the volatility, becomes the dominant factor in valuation. For this reason, the Shelton model has performed reasonably well over the years for in-the-money warrant price prediction.

Dividends that might be paid during the option's life will affect the option price. Merton (1973) adjusted the Black-Scholes model for dividends (leakage) on a continuous basis by reducing the stock price for the forgone dividends:

$$C_E = e^{-\delta(t)} SN(d_1) - Xe^{-r(t)} N(d_2),$$
$$d_1 = ln\ (S/X) + (r - \delta + .5\sigma^2)(t) / \sigma\sqrt{t},$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

Dividend yield on the stock is transformed to the continuous leakage rate by:

$$\delta = ln\ (1+D),$$

Where:

D = *annual dividend yield.*

Likewise, the risk-free yield-to-maturity interest rate is transformed to the continuous annual rate by:

$$r = ln\ (1 + YTM),$$

Where:

YTM = *annual yield to maturity rate.*

Dividends may add value to the American option's early exercise privilege. To value the American style option investor's early exercise privilege, an analytical approximation by Barone-Adesi and Whaley (1987) can be applied to Merton's continuous dividend European call option model. A heuristic rule exists that states: Never exercise an American style option early unless the anticipated dividend yield to be received during the option's life exceeds the risk-free interest rate. An investor may sell the option early, but otherwise should not exercise it early. Although employee stock options generally cannot be sold, Huddart (1994), and Kulatilaka and Marcus (1994) argue that the option holder risk aversion level may reduce option values and, accordingly, induce the early exercise of the option. In contrast, Carpenter (1998) finds that executives hold options long enough and deep enough into the money before exercising.

The investor's choice in the early exercise or not decision depends on the dividend rate, the risk-free interest rate, and the time to expiration. If the stock price reaches some critical level S^*, the decision to capture $S_t - X$ should be made. If the stock price is below S^*, then the investor should hold the option until expiration. Accordingly, for any stock price greater than S^*, the European call (no early exercise privilege) will be worth less than the exercisable proceeds for the American call. Therefore, the American call investor is indifferent about exercise at stock price S^*, when both the American and European call prices are worth the same, $S^* - X$. For higher stock prices (above S^*) the value of the European call falls below that of the American call, and the value of the American call becomes equal to its exercisable proceeds. The American call option investor preferring more wealth to less, will exercise the capture of $S_t - X$ and will invest those proceeds to the expiration date of the call option to earn a return that would be lost if the option were not exercised.

The critical stock price is determined by the following iterative process:

$$S^* - X = C_E + \{1 - e^{\delta t} N(d_1)\} (s^*/q_2),$$

Where:

C_E is priced at S^*,
C_A = the price of an American Call Option,
$N(d_1)$ is evaluated at S^*,
$q_2 = 1 - m + \sqrt{\ } (m-1)^2 + 4K / 2$,
$m = 2(r - \delta) / \delta^2$
$K = 2r / [\delta^2 (1 - e^{-r(t)})]$,
$A_2 = S^*[1 - e^{-\delta(t)} N(d_1)] / q_2$,
$C_A = C_E + A_2 (s/s^*)^{q2}$ if $S < S^*$,
$C_A = S - X$, if $S \geq S^*$.

VII. Prior Warrant Pricing Studies

Pratt (1989) undertook an empirical investigation of the Shelton, Kassouf, and Noreen and Wolfson warrant price prediction models. The Shelton model was easy to explain, and, if it accurately predicted warrant prices, it would be a useful model for business appraisers. The Kassouf (1965) model requires extensive empirical estimation.

For a sample of 25 warrants on December 15, 1986, Pratt estimated warrant prices using each of the three models.

He tested the Shelton model using both the *L=1* method and the *L=0* method. His results indicated that the Shelton model with the *L* traded option variable *(L=0)* predicted warrant values that were significantly closer to actual market prices than when the *L=1* variable was included. The Kassouf model had the most accurate warrant price prediction ability. The difficulty of using the Kassouf model (estimating the regression coefficients and the underlying stock price volatility) seems to recommend the Shelton model as a relatively accurate model to estimate warrant prices.

The Noreen and Wolfson (1981) expanded on the Black-Scholes model and adapted it to the valuation of employee stock options. They argued that warrants were similar to employee stock options in many aspects including long-term to expiration and that exercise of either results in issuance of additional stock. The Noreen-Wolfson model uses Merton's continuous dividend European call option model by making the following adjustment to the predicted call price:

$$W = \frac{N}{N + M} C_E ,$$

Where:

W = warrant price,
N = number of common shares outstanding,
M = number of common shares to be issued if warrants are exercised

Hauser and Lauterbach (1997) argue that the Black-Scholes option-pricing model should be adjusted for the dilution of earnings per share and the exercise price cash inflow to the company caused by the potential exercise of the outstanding warrants. In addition to the Noreen-Wolfson adjustment stated previously, they also adjust the stock price, *S*, by adding the value of the total outstanding warrants divided by the number of common shares outstanding. The dilution adjusted Black-Scholes option model approach reduces the predicted option price, as does the constant elasticity of variance model approach. Perhaps it is possible that investors do not place great weight on the volatility measure of the underlying stock when the warrant is in-the-money with a long-term to expiration. Hauser and Lauterbach do not explain the effect on security prices when fully diluted earnings per share have been adjusted for in-the-money stock options and warrants. Haven't investors already adjusted the security price for the potential exercise of the stock option or warrant?

Although many warrant pricing models reduce the predicted warrant value by a dilution adjustment, this adjustment is not necessary based on both theoretical and empirical grounds. If investors believe the warrants will be exercised, then the stock price will react at the warrant issue announcement date. Therefore, no adjustment is necessary since the stock price reaction has already occurred (Galai 1989 and Sidenius 1996). The dilution adjustment proposed by Noreen and Wolfson is already accounted for in the stock price, which reflects the potential future exercise of the warrants (Crouhy and Galai 1991).

In 1991, Barenbaum and Schubert made an empirical examination of both the Black-Scholes and Shelton models. Their July 17, 1989 sample consisted of 58 warrants.

In testing the Shelton model, they reported only the $L=1$ option adjustment approach. For the Black-Scholes model, the stock price volatility measure was determined using the last five years of monthly stock prices and the risk-free rate was Treasury issue yields matching the warrant maturity. They applied the mean absolute error (MAE) statistic to Pratt's study and concluded that the Black-Scholes model had a lower MAE (21.6%) than the Shelton model (28.9%).

Barenbaum and Schubert further subgrouped their sample into both in-the-money and out-of-the-money warrants. Their testing indicated that the Shelton model consistently overvalued out-of-the-money warrants; whereas, the Black-Scholes model overvalued 43 percent of the warrants tested. The Shelton model with the $L=1$ adjustment resulted in an overvaluation bias for out-of-the-money warrants. For at or in-the-money warrants, both the Black-Scholes and the Shelton models valued warrants with similar accuracy when measured by mean absolute error. Both models had a tendency to overvalue the in-the-money warrants. The Black-Scholes model, however, had a significantly lower median absolute error. In contrast to Pratt's study, although both models had significant pricing errors, Barenbaum and Schubert found the Black-Scholes option-pricing model superior to the Shelton model.

Pratt, and Barenbaum and Schubert each conducted empirical tests using versions of the Black-Scholes and Shelton models that resulted in conflicting conclusions. Neither study considered adjusting the Black-Scholes model for the value of the early exercise privilege inherent in the warrant price. This study will attempt to resolve the conflict of the prior studies and will attempt to further the research into this area by considering the use of a constant volatility measure and a linear model for warrant pricing.

VIII. Primary Research Questions:

1. H_0 There is no difference in the warrant price predicted by the Black-Scholes model and the market warrant price.
   H_1: The Black-Scholes model does not properly predict market warrant prices.

2. H_0: There is no difference in the warrant price predicted by the Shelton model *(L=0)* and the market warrant price.
   H_1 The Shelton model *(L=0)* does not properly predict market warrant prices.

IX. Methodology

A sample of 68 warrant market prices and their underlying stock market prices, which are presented in TABLE 1, were obtained from those warrants listed in the Value Line Convertibles Survey during the period of August 18, 1997 to October 13, 1997. Of the 79 warrants listed, 68 warrants were also listed in Omega Research's October 1, 1997 CD-ROM. The Omega Research CD-ROM was used to obtain closing stock price and warrant price data as well as trading volume. The risk-free interest rate was obtained from the Wall Street Journal for yields to maturity on a zero coupon Treasury matching each warrant's term to expiration. Underlying stock return volatility was calculated using annualized monthly average daily closing stock prices for the historical period prior to the observation date that matched the term to expiration of the warrant. If the term to expiration was less than one year, the annualized daily stock return volatility for the seven months prior to the observation date was used. No observations were omitted from the analysis.

Both the Merton European continuous dividend call option model and the analytical approximation American continuous dividend versions of the Black-Scholes model were tested. The Summary Error Statistical Differences (Exhibit 1) for the model predicted warrant price and the market warrant price, consisting of the mean error (ME), the mean absolute error (MAE), and the mean absolute percentage error (MAPE), are presented below for the sample of 68 warrants and the subgroups of at or in-the-money warrants (35 warrants), and out-of-the-money warrants (33 warrants):

Exhibit 1
Summary Error Statistical Difference

	SIZE	ME	MAE	MAPE
Total Sample Warrants				
Black-Scholes:European	68	0.51	1.04	41.03%
Black Scholes:American	68	0.51	1.05	40.99
Shelton:L=1	68	1.21	1.74	246.55
Shelton:L=0	68	0.59	1.15	170.57
At or In-the Money Warrants				
Black-Scholes:European	35	1.15	1.48	17.91%
Black Scholes:American	35	1.16	1.48	17.84
Shelton:L=1	35	1.36	2.27	24.78
Shelton:L=0	35	0.72	1.52	17.38
Out-of-the-Money Warrants				
Black-Scholes:European	33	-0.17	0.58	65.56%
Black Scholes:American	33	-0.17	0.58	65.55
Shelton:L=1	33	1.06	1.18	481.75
Shelton:L=0	33	0.45	0.75	333.10

All the models analyzed in Exhibit 1 indicated an overpricing bias for the sample and the subgroups, with the exception of the Black-Scholes models for the subgroup out-of-the-money warrants. The mean error is useful in assessing bias in the prediction model. The higher the mean error, the greater the systematic deficiency in the model prediction. The mean absolute error measures the dispersion of the prediction errors and it's measure is determined without regard to whether the error was an overestimate or an underestimate. Ideally, a prediction model should have no bias and no error dispersion. The user must decide which error measurement parameter is most important. Normally, preference is given to lower values of mean absolute error.

Consistent with Pratt's findings, the Shelton unadjusted model *(L=0)* outperformed the Shelton adjusted model *(L=1)* in terms of lower mean error and mean absolute error. The Shelton unadjusted model *(L=0)* slightly outperformed the Black-Scholes model in terms of MAPE for the subgroup at or in-the-money warrants. This result is in contrast to Barenbaum and Schubert's findings that the European version Black-Scholes model outperformed the Shelton adjusted model *(L=1)*. Both the Black-Scholes models, however, outperformed the Shelton models in terms of the mean error and the mean absolute errors for the sample and the subgroup out-of-the-money warrants. The Black-Scholes models appear to be the models of choice when valuing out-of-the-money warrants. This conclusion is consistent with Barenbaum and Schubert's findings. The differences in predicted warrant prices between the European and American versions of the Black-Scholes model are slight due to the few number of stocks paying dividends (sample of nine warrants). Intuitively, the American version Black-Scholes model should have outperformed the European model in the prediction of warrant prices.

X. Matched-paired Samples *T-Test* Of Mean

This procedure is used to compare two sets of data collected from two different populations when the observations in the two samples are paired. The paired differences of the two samples are treated as if they were a single sample from a population of differences using the *T-test* of sample means. If the two samples are independent, the variance of the pair-wise differences would be close to the sum of the variances of the two samples. If the variance of the differences is different, then the two samples are correlated.

The *T-test* of paired samples assumes a normal distribution of errors. By invoking the Central Limit Theorem, a large sample (over 30 observations) may be assumed to have a normal distribution. The computed *T* statistic tests the hypothesis that the mean difference between the model predicted warrant price and the market warrant price is equal to zero. If the computed P-value is less than 0.05, the null hypothesis is rejected and the alternative hypothesis is accepted. In a paired comparison, interest typically centers on the mean differences. If the value of 0.0 lies between plus and minus one standard deviation of the mean difference, then the null hypothesis cannot be rejected.

Exhibit 2
Matched-Pair T Test

	T Statistic	P Value	Mean Market Warrant Price	Mean Model Warrant Price	Mean Difference	Standard Deviation Difference	Sample Size
Total Sample Warrants							
Black-Scholes European	2.04	.04	11.63	12.13	.50	2.04	68
Black-Scholes:American	2.08	.04	11.63	12.14	.51	2.04	68
Shelton:L=1	4.04	.00	11.63	12.84	1.21	2.48	68
Shelton.L=0	2.77	.01	11.63	12.22	.59	1.76	68
At or In-the-Money Warrants							
Black-Scholes·European	2.64	.01	21.41	22.56	1.15	2.57	35
Black-Scholes.American	2.68	.01	21.41	22.57	1.16	2.56	35
Shelton L=1	2.68	.01	21.41	22.77	1.36	2.99	35
Shelton·L=0	2.07	.04	21.41	22.14	.72	2.08	35
Out-of-the-Money Warrants							
Black-Scholes European	-1.15	.26	1.07	1.07	-.17	.87	33
Black-Scholes American	-1.15	.26	1.07	1.07	-.17	.87	33
Shelton:L=1	3.36	.00	2.31	2.31	1.06	1.81	33
Shelton·L=0	1.89	.07	1.69	1.69	.45	1.36	33

In Exhibit 2, both the T-test statistic and the paired mean difference tests reject the null hypothesis that the mean of the model predicted warrant price and market warrant price difference equals zero for the sample and the subgroup at or in-the-money warrants. The null hypothesis, however, is not rejected for the Black-Scholes models and the unadjusted Shelton model *(L=0)* for the subgroup out-of-the-money warrants. Both the sample and the subgroup's normality hypothesis tests were rejected. Therefore, the null hypothesis that there is no significant difference in means was further tested using the Wilcoxon matched-pairs signed rank test.

XI. Wilcoxon Matched-Pairs Signed Rank Test

A non-parametric test called the Wilcoxon matched-pairs signed rank test was developed for situations where the decision maker has related samples (market warrant price and model predicted price) and is unable to use the paired sample *T-test*. The Wilcoxon matched-pairs signed rank tests the null hypothesis that there is no difference in the means (Groebner and Shannon 1993). The Wilcoxon matched-pairs signed rank test uses the information about the size of the difference among the paired data. It is more likely to detect true differences when they exist. The test does require that the differences be a sample from a symmetric distribution. If the *P-value* is less than the alpha level of 0.05, then the null hypothesis of no difference in means between the model predicted warrant price and the market warrant price is rejected.

Exhibit 3
Wilcoxon Matched-Pairs Signed Rank Test

	Z Test Statistics	P Value
Total Sample Warrants		
Black-Scholes:European	-1.83	.07
Black-Scholes:American	-1.90	.06
Shelton:L=1	-4.98	.00
Shelton:L=0	-3.21	.00
At or In-the-Money Warrants		
Black-Scholes:European	-3.23	.00
Black-Scholes:American	-3.31	.00
Shelton:L=1	-3.23	.00
Shelton:L=0	-2.26	.02
Out-of-the-Money Warrants		
Black-Scholes:European	-1.08	.28
Black-Scholes:American	-1.08	.28
Shelton:L=1	-4.01	.00
Shelton:L=0	-2.28	.02

As stated in Exhibit 3, the Wilcoxon matched-pairs signed rank test for the Shelton models of the total sample rejected the null hypothesis. The null hypothesis was also rejected for both Shelton models for the subgroup out-of-the-money warrants and the null hypothesis was rejected for all models for the subgroup at or in-the-money warrants.

Results for the subgroup of at or in-the-money warrants are in contrast to Barenbaum and Schubert's findings that indicate the European version Black-Scholes model outperformed the adjusted Shelton *(L=1)* model.

In testing the Black-Scholes model, MacBeth and Merville (1979) found the following systematic discrepancies between predicted and market option prices:

1. Black-Scholes model predicted prices tended to be higher than market prices for out-of-the-money options.
2. Black-Scholes model predicted prices tended to be less than market prices for in-the-money options.

This study, in contrast to MacBeth and Merville's findings, reaches exactly opposite conclusions for the Black-Scholes model predicted price for long-term warrants.

Stock return volatility may be estimated in two basic ways. Historical data of the stock's recent past or similar option market comparables may be used. An estimate of the stock's standard deviation obtained from the options market is referred to as implied volatility. The correlation between the implied volatility and the historical volatility for the sample and the subgroups indicate little relationship, as shown in Exhibit 4. Only the Spearman rank correlation for the subgroup out-of-the-money warrants indicated a relationship significantly different from zero at the 5% level. The Spearman rank correlation coefficients are computed between the ranks of the data, rather than between the matched-pairs data. These coefficients are less affected by outliers or non-normal distributions.

Exhibit 4
Correlation of Implied and Historical Volatility

	Correlation	P Value	Spearman Rank	P Value
Total Sample	.1945	.1120	.2336	.0559
At or In-the-Money	-.0675	.7001	.0236	.8905
Out-of-the-Money	.3325	.0587	.4456	.0117

XII. Linear Approach For At or In-the-Money Warrants

All the models rejected the null hypothesis that there was no difference in the means between the model predicted warrant price and the market warrant price. The models consistently overvalued the warrant prices. In-the-money warrants generally have warrant deltas that approach *1* $(N(d_1)=1)$.

Within the upper and lower option price boundaries lies a convex curve that represents the option's value as a function of the stock price. The option value decreases as the time to expiration decreases. In *Figure 1*, the option value is presented when the time to expiration, t, is ten times the otherwise identical option when $t = 1$. Most of the option value line curvature appears where the stock price is less than the exercise price, *X*. When the stock price exceeds the exercise price $(S > X)$ the curvature of the option-pricing line decreases to approximately a linear relationship. If the assumptions that either the stock has little risk, or if there is little time left on the option are made, then $N(d_1) \approx N(d_2) \approx 1.0$ and the European dividend adjusted version of the Black-Scholes model becomes:

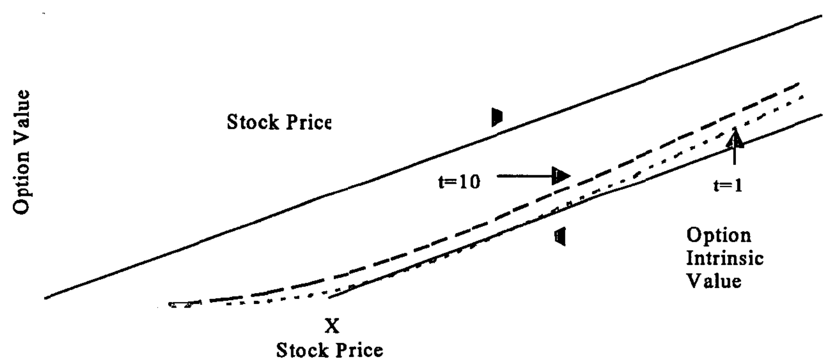
$$W = Se^{-\partial t}(1) - Xe^{-rt}(1)$$

Where:

W = warrant price.

At this point, the warrant value approximates the current stock price reduced by the present value of the foregone dividends less the present value of the exercise price. The warrant price computed by this approach (the exclusion of volatility in estimating the option value) is called the minimum value for nonpublic entities, as stated in FASB 123.

Figure 1
Stock-Option Relationship



The estimated product moment correlation for the subgroup at or in-the-money warrants indicates a 0.9971 correlation with the market warrant price with a *P-value* of 0.00. The Spearman rank correlation was 0.9886 with a *P-value* of 0.00. Therefore, the paired variables of adjusted intrinsic value and market warrant price are significantly correlated at the 5% level.

This strong correlation relationship allows for the development of a linear regression model to predict warrant prices for at or in-the-money warrants. A linear regression of the subgroup at or in-the-money warrants indicates the following relationship for market warrant price:

$$W = -0.031926 + 0.990659\,(Se^{-\partial t} - Xe^{-rt}).$$

Exhibit 5
Table of Estimates

	Estimate	Standard Error	T Value	P Value
Intercept	-.031926	.383845	-.08	.9342
Slope	.990659	.0131228	75.49	.0000

R-Squared = 99.42%
Correlation Coefficient = .997
Standard Error of Estimation = 1.52742
Durbin-Watson Statistic = 1.58056
Mean Absolute Error = 1.05874
Sample Size = 35

The above regression, as shown in Exhibit 5, indicates a statistically significant relationship between the adjusted intrinsic value of the warrant and market warrant price at the 5% significance level, since the *P-value* of the slope is less than 5%. The model explains 99.42% of the variability in market warrant price, as indicated by the *R- squared* value.

Exhibit 6
Analysis of Variance

Source	Sum of Squares	D.F.	Mean Square	F Ratio	P Value
Model	13295.7	1	13295.7	5698.96	.0000
Error	76.9893	33	2.33301		

Exhibit 7
Matched-Pair T-Test

	T Statistic	P Value	Mean Market Warrant Price	Mean Model Warrant Price	Mean Difference	Standard Deviation Difference	Sample Size
At or In-the-Money Warrants							
Linear Approach							
Sample	- 036	971	21 41	21 42	009	1 504	35
Barenbaum & Schubert Sample	1 238	237	13 34	12 49	- 848	2 563	14
Pratt Sample	1 831	100	4 16	3 23	- 934	1 613	10
Constant Volatility - 20.2%							
Sample	-1 451	155	21.41	21.76	348	1 422	35
Barenbaum & Schubert Sample	444	664	13 34	13 06	- 281	2 371	14
Pratt Sample	- 046	963	4 17	4 14	- 02	1.525	10
Out-of-the-Money Warrants							
Constant Volatility - 46.5%							
Sample	1 544	132	1 24	1 02	- 223	830	33
Barenbaum & Schubert Sample	-2 754	008	1 20	2 27	1 075	2 531	42
Pratt Sample	-2 069	057	4 71	6.13	1 416	2.651	15

XIII. Constant Long-Term Market Volatility

Since all the original models tested for the subgroup at or in-the-money warrants overstated the warrant price, investors may be using lower expected future volatility measures in the pricing of in-the-money warrants. Alternatively, when the stock price (*S/X*) and time-to-maturity are relatively large, the implied volatility becomes relatively small. Accordingly, instead of using the underlying stock return's historical standard deviation as the measure of volatility, the long-term, New York Stock Exchange Index volatility for the years 1926 to 1996 or 20.2 percent (Ibbotson Associates 1997 Yearbook) is used as the volatility measure in the American version Black-Scholes model. The constant volatility measure is further tested using both Pratt's, and Barenbaum and Schubert's samples as shown in Exhibits 7, 8, and 9.

A constant volatility of 20.2% in the American version Black-Scholes model failed to reject the null hypothesis for both the matched-pair *T*-test and the Wilcoxon matched-pairs signed rank test. Accordingly, there was no statistically significant difference in the means between the model predicted warrant price and the market warrant price.

XIV. Out-of-the-Money Warrants

The out-of-the-money warrant price prediction is more difficult since volatility of the underlying stock becomes the dominant factor in valuing the warrant.

Exhibit 8
Wilcoxon Matched-Pairs Signed Rank Test

	Z Test Statistic	P Value
At or In-the-Money Warrants		
Linear Approach		
Sample	-.409	.682
Barenbaum & Schubert Sample	-1.538	.124
Pratt Sample	-1.580	.114
Constant Volatility - 20.2%		
Sample	-1.450	.147
Barenbaum & Schubert Sample	-.973	.331
Pratt Sample	-.357	.721
Out-of-the-Money Warrants		
Constant Volatility - 46.5%		
Sample	-1.313	.189
Barenbaum & Schubert Sample	-1.494	.135
Pratt Sample	-1.931	.053

Exhibit 9
Summary Error Statistical Differences

	Size	ME	MAE	MAPE
At or In-the-Money Warrants				
Linear Approach				
Sample	35	.01	1.06	12.52%
Barenbaum & Schubert Sample	14	.85	1.94	23.18
Pratt Sample	10	-.94	1.33	35.40
Constant Volatility - 20.2%				
Sample	35	.35	1.05	11.34%
Barenbaum & Schubert Sample	14	-.28	1.57	18.59
Pratt Sample	10	-.02	1.07	30.00
Out-of-the-Money Warrants				
Constant Volatility - 46.5%				
Sample	33	-.22	.54	64.63%
Barenbaum & Schubert Sample	42	1.08	1.28	137.71
Pratt Sample	15	1.42	1.59	34.02

A constant volatility approach was used to predict warrant prices as shown in Exhibit 9. The historical standard deviation for the years 1926 to 1996 of 46.5% for the New York Stock Exchange 10th decile as reported by Ibbotson Associates was selected for testing. The American version Black-Scholes model was used for the current sample as well as Pratt's, and Barenbaum and Schubert's samples.

The null hypothesis that there is no difference in the means between the model predicted warrant price and the market warrant price for all samples could not be rejected as shown in Exhibits 7 and 8. However, the Barenbaum and Schubert sample for the matched-pair *T*-test (Exhibit 7) did reject the null hypothesis. Since this sample also rejected the matched-pair normality hypothesis, I have relied on the Wilcoxon matched-pairs signed rank test (Exhibit 8) which failed to reject the null hypothesis.

The linear and the constant volatility Black-Scholes models for the at or in-the-money warrants subgroup indicates statistical significance in the prediction of warrant prices across time periods. Researchers often examine the deficiencies and biases of an option-pricing model by using univariate regressions of the pricing error on various parameters, such as time to expiration, stock return volatility, and stock price divided by exercise price (Whaley 1982). Such an analysis was performed in this study for the data shown in Exhibit 10. The Summary of Average Pricing Errors By Time to Expiration appears to indicate that the average MAPE for the linear model subgroup at or in-the-money warrants increases with time to expiration. But the univariate regression of the pricing error on the variables time to expiration, stock return volatility, and stock price divided by exercise price failed to indicate a statistically significant relationship. The constant volatility Black-Scholes model indicated mixed MAPE results with respect to time to expiration. Again, there was no statistically significant relationship in the pricing error to time to expiration in the univariate regression. This implies that business appraisers should be able to use a constant volatility Black-Scholes model or a linear model for valuing in-the-money employee options.

Exhibit 10
Summary of Average Pricing Errors by Time to Expiration

		Model Average MAPE		
Time to Expiration	**# of Warrants**	**Linear**	**Constant Volatility 20.2%**	**Constant Volatility 46.5%**
At or In-the-Money Warrants				
Less Than One Year				
Sample	11	8.68	6.80	-
Barenbaum & Schubert Sample	-	.65	.51	-
Pratt Sample	-	-	-	-
One to Two Years				
Sample	11	13.67	12.10	-
Barenbaum & Schubert Sample	7	14.96	9.34	-
Pratt Sample	1	62.13	50.89	-
Over Three Years				
Sample	13	15.19	15.19	-
Barenbaum & Schubert Sample	6	36.54	32.40	-
Pratt Sample	9	29.22	27.67	-
Out-of-the-Money Warrants				

Time to Expiration	# of Warrants	Model Average MAPE: Linear	Constant Volatility 20.2%	Constant Volatility 46.5%
Less Than One Year				
Sample	5	-	-	62.98
Barenbaum & Schubert Sample	20	-	-	113.82
Pratt Sample	-	-	-	-
One to Two Years				
Sample	18	-	-	63.97
Barenbaum & Schubert Sample	11	-	-	124.29
Pratt Sample	2	-	-	90.63
Over Three Years				
Sample	10	-	-	65.97
Barenbaum & Schubert Sample	11	-	-	86.56
Pratt Sample	13	-	-	25.31

XV. Summary and Conclusion

Stock price *(S/X)* correlation with volatility maintains a constant state of change between a positive and negative relationship. When volatility is positively correlated with stock price *(S/X)*, high stock price is associated with high volatility. As the stock price rises, the probability of large positive changes increase. If stock price falls, it becomes less likely that large changes take place. When volatility is negatively correlated with stock price *(S/X)*, the reverse is true. Price increases reduce the volatility; therefore, it is unlikely that very high stock prices will result. Stock price decreases increase volatility, increasing the change of large positive price changes and very low prices become less likely.

Because the Black-Scholes model price is approximately linear with respect to volatility, increasing the time-to-maturity with all else held constant will result in a similar effect as increasing volatility. Longer term warrants have lower implied volatilities, as determined by the Black-Scholes model, than do shorter term warrants whenever the Black-Scholes price overprices the warrant. As the sample stock price *(S/X)* increased, the implied volatility decreased.

Any increase in the stock price *(S/X)* and the time-to-maturity *(t)* that causes the option delta to approach one will cause the implied volatility determined by the Black-Scholes model to become relatively low. The option delta cannot exceed one. Accordingly, business appraisers need not be overly concerned about the volatility measure for the valuation of at or in-the-money long-term options. The FASB 123 minimum value method, or a linear model, or a low volatility input measure to the Black-Scholes model will result in relatively accurate employee option price predictions.

Use of historical volatility measures as a substitute for expected future volatility of the underlying stock will not result in reasonably accurate employee option estimate predictions. Accordingly, both FASB 123 and Internal Revenue Procedure 98-34 recommendations in the use of historical volatility measures as a substitute for future expected volatility will result in unreasonable employee stock option price predictions. The business appraiser is just as well served to use a volatility measure of approximately 46.5% as the input to the Black-Scholes model in valuing long-term out-of-the-money employee stock options. Finally, business appraisers should not use the Shelton model to value employee stock options as other models outperform it in option price prediction.

Appendix

TABLE 1

TOTAL SAMPLE 68 WARRANTS

STOCK SYMBOL	DATE	STK PR	WARR PR	EXER PR	CONV	ADJ WARR PR	YRS TO EXPI	YTM	DIV YLD	STD DEV
ADSO	8/15/97	0.66	0.13	5.36	1.26	0.10	1.22	0.058		0.61
AES	8/13/97	78.38	52.63	29.43	1.00	52.63	2.96	0.059		0.31
AIS	8/22/97	5.75	0.03	16.00	1.00	0.03	1.44	0.056	0.036	0.26
ALO	8/15/97	19.00	4.13	21.94	1.00	4.13	1.46	0.056	0.010	0.39
AMES	8/15/97	13.25	12.13	1.11	1.00	12.13	1.39	0.056		0.84
ANCO	8/13/97	13.13	5.38	12.25	1.00	5.38	3.81	0.061		0.41
AQUX	8/13/97	6.94	1.63	6.00	1.00	1.63	2.08	0.058		0.38
ASYS	8/14/97	2.75	1.25	3.25	1.00	1.25	2.34	0.059		0.46
AWA	8/15/97	12.75	4.63	12.74	1.00	4.63	2.03	0.058		0.47
BJS	8/15/97	64.13	38.88	30.00	1.00	38.88	2.66	0.059		0.36
CXC	8/14/97	39.88	150.00	2.13	4.00	37.50	0.08	0.054		0.42
CXI	8/15/97	4.44	0.94	8.40	1.00	0.94	3.87	0.061		0.55
DSGR	8/22/97	3.38	0.44	5.50	1.00	0.44	2.21	0.059		0.60
FLT	8/15/97	64.81	24.00	43.88	1.00	24.00	3.45	0.060	0.027	0.21
FPX	8/22/97	1.69	0.50	3.75	1.00	0.50	1.10	0.057		0.56
GYM	8/13/97	7.94	0.06	20.00	1.25	0.05	1.34	0.058		0.49
IBET	8/6/97	0.68	0.06	8.50	1.00	0.06	2.35	0.059		1.00
INTC	8/15/97	92.13	72.00	20.88	1.00	72.00	0.57	0.056		0.72
KYZN	8/18/97	1.38	0.31	5.00	1.00	0.31	2.96	0.059		0.76
LCE	8/15/97	51.19	34.00	18.75	1.00	34.00	3.38	0.060	0.004	0.26
LFUS	8/20/97	29.38	25.00	4.18	1.00	25.00	4.35	0.061		0.25
LJPC	8/15/97	4.75	0.75	12.00	0.50	1.50	1.80	0.058		0.65
LSR	8/15/97	3.44	0.06	6.00	1.00	0.06	0.41	0.056		0.26
LTV	8/15/97	12.81	0.31	16.82	0.56	0.56	0.87	0.056		0.31
MCHM	8/15/97	6.06	2.88	3.00	1.00	2.88	0.33	0.056		0.64
MDN	8/15/97	22.38	6.50	16.23	1.00	6.50	1.52	0.057	0.052	0.21
NIAG	8/15/97	7.13	1.81	5.50	1.00	1.81	3.99	0.061		0.83
ORTC	8/15/97	9.75	1.38	15.00	1.00	1.38	1.43	0.057		0.53
ORYX	8/13/97	0.97	1.00	1.84	1.90	0.53	1.65	0.058		0.92
OXGN	8/15/97	39.88	29.00	11.54	1.07	27.10	1.03	0.057		0.54
PCTH	8/15/97	4.06	2.00	4.69	1.00	2.00	3.92	0.061		0.67
PMOR	8/15/97	7.13	1.63	13.50	1.00	1.63	5.07	0.061		0.33
POSI	8/15/97	0.81	0.19	8.25	1.00	0.19	1.34	0.058		1.28
QDEL	8/15/97	4.13	1.13	7.50	1.00	1.13	4.71	0.061		0.50
SCIO	8/15/97	7.56	0.88	26.74	1.00	0.88	0.88	0.056		0.35
TLMD	8/15/97	31.00	23.13	7.00	1.00	23.13	2.36	0.059		0.39

TABLE 1 Continued

TOTAL SAMPLE 68 WARRANTS

STOCK SYMBOL	DATE	STK PR	WARR PR	EXER PR	CONV	ADJ WARR PR	YRS TO EXPI	YTM	DIV YLD	STD DEV
TRV	8/15/97	65 88	94.75	19 50	2.00	47 38	0.96	0 06	0 009	0.64
TWA	8/15/97	7 19	3 88	14 40	1.00	3.88	4.98	0 06		1 55
TWHH	8/15/97	8.13	2.13	6 50	1.00	2 13	0 31	0.06		0.21
UBS	8/15/97	9.63	1.75	18.40	0 50	3 50	0.69	0 06		0.52
USG	8/15/97	44.69	28.63	16 14	1.00	28 63	0 72	0.06		0 34
VISN	8/14/97	4 31	1.19	6.00	1.00	1.19	2 03	0 06		0.44
WANG	8/15/97	20.31	6.00	21.45	1 00	6 00	2 85	0.06		0.47
WONE	8/15/97	30 63	13.25	17.25	1 00	13.25	0.05	0 05		0 80
YILD	4/29/97	6 88	1 50	11.00	1 00	1 50	3 56	0 06		0 72
FBS	6/18/97	86 63	71 00	6.25	0 90	78.89	3 42	0.06	0.021	0.17
NEO	8/15/97	4.50	2.50	4 90	2.00	1.25	3.45	0.06		0.81
RLCO	8/5/97	3 50	0.31	8.40	1.00	0 31	3 49	0.06		0.43
ZNRG	8/15/97	4.75	1 56	5.50	1 00	1.56	3 33	0.06		0.42
ARSN	8/15/97	9 63	1 53	7 50	0.50	3.06	0 56	0 06		0 50
AZ	8/15/97	0.38	0.13	15.63	1 00	0.13	10.00	0.06		0 62
KE	8/14/97	19.44	11.38	8 00	1.00	11 38	1 88	0.06	0.010	0 20
UH	8/15/97	32 69	13.75	20 00	1 00	13 75	0.85	0 06		0 34
TTRR	8/15/97	29.38	27.00	2.54	1.00	27.00	4.37	0 06		0.38
GNV	8/14/97	3.56	1 00	11.00	1 00	1.00	2.59	0.06		0.48
BNO	8/21/97	14.25	5.00	11.00	1 00	5 00	1 41	0 06		0 42
BGN	8/15/97	4.50	0 75	5.50	1 00	0.75	2 98	0.06		0.50
BGLV	8/14/97	51.00	39.44	10 00	1.00	39.44	3 01	0 06		0.38
ATCS	8/12/97	10.94	1 88	10.00	1.00	1.88	0 71	0.06		0.52
AZA	8/18/97	29.44	0.19	65 00	0.13	1.50	2.37	0.06		0 28
SILCF	8/25/97	6.25	0 94	7 50	1.00	0 94	0.48	0 06		1.19
JCOR	8/12/97	43.38	4 38	28.00	0.20	21.45	4 10	0 06		0 62
AMV	8/18/97	22.38	7.75	16.42	1.00	7.75	4.62	0.06	0 006	0 27
GSB	8/15/97	28 63	17 00	12.00	1 00	17 00	3.02	0 06		0.52
MLD	8/11/97	4 00	2 00	4 00	1.00	2 00	5.22	0 06		0.85
ZLC	8/13/97	22.44	12.38	10.37	1.00	12.38	0 92	0.06		0.26
NGCO	9/19/95	53.75	38 88	14 50	1.00	38.88	4 78	0 06		0 37
NCS	12/6/95	26.88	9.00	17.75	1 00	9 00	2.26	0.06		1.47

TABLE 2
TOTAL SAMPLE 68 WARRANTS

STOCK SYMBOL	BS EURO	BS AMER	SHEL L= 1	SHEL L= 0
ADSO	0.00	0.00	0.29	0.21
AES	53.69	53.69	54.22	52.82
AIS	0.00	0.00	1.85	1.34
ALO	2.88	2.88	6.39	4.68
AMES	12.23	12.23	11.16	11.42
ANCO	5.43	5.43	6.00	4.64
AQUX	2.28	2.28	3.33	2.48
ASYS	0.72	0.72	1.04	0.77
AWA	3.88	3.88	4.67	3.43
BJS	38.95	38.95	41.42	39.48
CXC	37.75	37.75	36.01	36.47
CXI	1.31	1.31	1.91	1.40
DSGR	0.79	0.79	1.26	0.93
FLT	24.08	24.35	36.31	32.22
FPX	0.07	0.07	0.53	0.39
GYM	0.21	0.21	2.62	1.92
IBET	0.07	0.07	0.26	0.19
INTC	71.86	71.86	70.48	70.68
KYZN	0.30	0.30	0.55	0.41
LCE	35.11	35.12	35.74	34.86
LFUS	26.15	26.15	23.33	23.82
LJPC	0.54	0.54	1.69	1.24
LSR	0.00	0.00	0.84	0.62
LTV	0.00	0.00	3.78	2.77
MCHM	3.13	3.13	3.52	3.40
MDN	6.02	6.25	10.90	9.62
NIAG	4.90	4.90	3.77	3.20
ORTC	1.32	1.32	3.27	2.40
ORYX	0.29	0.29	0.34	0.25
OXGN	29.01	29.01	28.98	28.81
PCTH	2.08	2.08	1.75	1.29
PMOR	1.31	1.31	3.28	2.41
POSI	0.09	0.09	0.27	0.20
QDEL	1.33	1.33	1.86	1.37
SCIO	0.00	0.00	2.25	1.65
TLMD	24.89	24.89	23.62	23.72

TABLE 2 Continued
TOTAL SAMPLE 68 WARRANTS

STOCK SYMBOL	BS EURO	BS AMER	SHEL L= 1	SHEL L= 0
TRV	46.99	47.03	47.60	47.27
TWA	3.95	3.95	3.29	2.42
TWHH	1.74	1.74	2.99	2.63
UBS	0.20	0.20	2.69	1.98
USG	29.17	29.17	30.42	29.92
VISN	0.74	0.74	1.58	1.16
WANG	7.04	7.04	8.11	5.96
WONE	13.42	13.42	15.27	14.77
YILD	3.02	3.02	2.90	2.13
FBS	75.54	75.54	71.82	74.10
NEO	2.59	2.59	1.88	1.38
RLCO	0.42	0.42	1.47	1.08
ZNRG	0.21	0.21	1.97	1.45
ARSN	2.76	2.76	3.93	3.45
AZ	0.06	0.06	0.20	0.15
KE	11.88	11.88	12.94	12.54
UH	13.73	13.73	17.33	16.10
TTRR	27.41	27.41	24.00	24.75
GNV	0.22	0.22	1.39	1.02
BNO	4.96	4.96	6.56	5.68
BGN	1.46	1.46	1.81	1.33
BGLV	42.61	42.61	39.52	39.91
ATCS	2.54	2.54	3.67	2.94
AZA	0.48	0.48	11.20	8.23
SILCF	1.69	1.69	1.60	1.17
JCOR	27.80	27.80	25.36	22.71
AMV	10.13	10.14	12.44	10.71
GSB	19.45	19.45	19.24	18.54
MLD	2.87	2.87	1.85	1.36
ZLC	12.57	12.57	13.97	13.47
NGCO	43.16	43.16	39.97	39.72
NCS	21.44	21.44	14.65	13.18

TABLE 3
TOTAL SAMPLE 68 WARRANTS

	BS EURO			BS AMER			SHELTON: L=1			SHELTON: L=0		
STOCK		ABS	ABS%		ABS	ABS%		ABS	ABS%		ABS	ABS%
SYMBOL	ERR	ERR	ERR	ERR	ER	ERR	ERR	ERR	ERR	ERR	ERR	ERR
ADSO	-0.10	0.10	1.00	-0.10	0.10	1.00	0.19	0.19	1.91	0.11	0.11	0.11
AES	1.06	1.06	0.02	1.06	1.06	0.02	1.60	1.60	0.03	0.19	0.19	0.00
AIS	-0.03	0.03	0.99	-0.03	0.03	0.98	1.82	1.82	58.75	1.31	1.31	42.19
ALO	-1.24	1.24	0.30	-1.24	1.24	0.30	2.26	2.26	0.55	0.56	0.56	0.14
AMES	0.10	0.10	0.01	0.10	0.10	0.01	-0.96	0.96	0.08	-0.70	0.70	0.06
ANCO	0.06	0.06	0.01	0.06	0.06	0.01	0.62	0.62	0.12	-0.74	0.74	0.14
AQUX	0.66	0.66	0.40	0.66	0.66	0.40	1.70	1.70	1.05	0.85	0.85	0.52
ASYS	-0.53	0.53	0.43	-0.53	0.53	0.43	-0.21	0.21	0.17	-0.48	0.48	0.39
AWA	-0.75	0.75	0.16	-0.75	0.75	0.16	0.05	0.05	0.01	-1.19	1.19	0.26
BJS	0.08	0.08	0.00	0.08	0.08	0.00	2.54	2.54	0.07	0.61	0.61	0.02
CXC	0.25	0.25	0.01	0.25	0.25	0.01	-1.49	1.49	0.04	-1.03	1.03	0.03
CXI	0.37	0.37	0.40	0.37	0.37	0.40	0.97	0.97	1.04	0.46	0.46	0.49
DSGR	0.35	0.35	0.80	0.35	0.35	0.80	0.82	0.82	1.88	0.49	0.49	1.12
FLT	0.08	0.08	0.00	0.35	0.35	0.01	12.31	12.31	0.51	8.22	8.22	0.34
FPX	-0.43	0.43	0.86	-0.43	0.43	0.86	0.03	0.03	0.06	-0.11	0.11	0.22
GYM	0.15	0.15	3.07	0.15	0.15	3.07	2.57	2.57	50.96	1.87	1.87	37.16
IBET	0.01	0.01	0.13	0.01	0.01	0.13	0.20	0.20	3.10	0.13	0.13	2.01
INTC	-0.14	0.14	0.00	-0.14	0.14	0.00	-1.52	1.52	0.02	-1.32	1.32	0.02
KYZN	-0.01	0.01	0.04	-0.01	0.01	0.04	0.24	0.24	0.77	0.09	0.09	0.30
LCE	1.11	1.11	0.03	1.12	1.12	0.03	1.74	1.74	0.05	0.86	0.86	0.03
LFUS	1.15	1.15	0.05	1.15	1.15	0.05	-1.67	1.67	0.07	-1.18	1.18	0.05
LJPC	-0.96	0.96	0.64	-0.96	0.96	0.64	0.19	0.19	0.12	-0.26	0.26	0.17
LSR	-0.06	0.06	1.00	-0.06	0.06	1.00	0.78	0.78	12.38	0.56	0.56	8.82
LTV	-0.56	0.56	1.00	-0.56	0.56	1.00	3.21	3.21	5.73	2.21	2.21	3.93
MCHM	0.26	0.26	0.09	0.26	0.26	0.09	0.65	0.65	0.23	0.53	0.53	0.18
MDN	-0.48	0.48	0.07	-0.25	0.25	0.04	4.40	4.40	0.68	3.12	3.12	0.48
NIAG	3.09	3.09	1.70	3.09	3.09	1.70	1.96	1.96	1.08	1.39	1.39	0.77
ORTC	-0.05	0.05	0.04	-0.05	0.05	0.04	1.89	1.89	1.38	1.03	1.03	0.75
ORYX	-0.24	0.24	0.45	-0.24	0.24	0.45	-0.19	0.19	0.36	-0.28	0.28	0.53
OXGN	1.90	1.90	0.07	1.90	1.90	0.07	1.88	1.88	0.07	1.71	1.71	0.06
PCTH	0.08	0.08	0.04	0.08	0.08	0.04	-0.25	0.25	0.12	-0.71	0.71	0.36
PMOR	-0.31	0.31	0.19	-0.31	0.31	0.19	1.65	1.65	1.02	0.78	0.78	0.48
POSI	-0.10	0.10	0.53	-0.10	0.10	0.53	0.08	0.08	0.43	0.01	0.01	0.05
QDEL	0.21	0.21	0.19	0.21	0.21	0.19	0.74	0.74	0.66	0.24	0.24	0.22
SCIO	-0.87	0.87	1.00	-0.87	0.87	1.00	1.37	1.37	1.57	0.77	0.77	0.88
TLMD	1.77	1.77	0.08	1.77	1.77	0.08	0.49	0.49	0.02	0.60	0.60	0.03

TABLE 3 Continued
TOTAL SAMPLE 68 WARRANTS

	BS EURO			BS AMER			SHELTON: L=1			SHELTON: L=0		
STOCK		ABS	ABS%		ABS	ABS%		ABS	ABS%		ABS	ABS%
SYMBOL	ERR	ERR	ERR	ERR	ER	ERR	ERR	ERR	ERR	ERR	ERR	ERR
TRV	-0.39	0.39	0.01	-0.35	0.35	0.01	0.23	0.23	0.00	-0.10	0.10	0.00
TWA	0.07	0.07	0.02	0.07	0.07	0.02	-0.58	0.58	0.15	-1.46	1.46	0.38
TWHH	-0.39	0.39	0.18	-0.39	0.39	0.18	0.86	0.86	0.41	0.50	0.50	0.24
UBS	-3.30	3.30	0.94	-3.30	3.30	0.94	-0.81	0.81	0.23	-1.52	1.52	0.44
USG	0.54	0.54	0.02	0.54	0.54	0.02	1.80	1.80	0.06	1.30	1.30	0.05
VISN	-0.45	0.45	0.38	-0.45	0.45	0.38	0.39	0.39	0.33	-0.03	0.03	0.02
WANG	1.04	1.04	0.17	1.04	1.04	0.17	2.11	2.11	0.35	-0.04	0.04	0.01
WONE	0.17	0.17	0.01	0.17	0.17	0.01	2.02	2.02	0.15	1.52	1.52	0.11
YILD	1.52	1.52	1.01	1.52	1.52	1.01	1.40	1.40	0.93	0.63	0.63	0.42
FBS	-3.35	3.35	0.04	-3.35	3.35	0.04	-7.07	7.07	0.09	-4.79	4.79	0.06
NEO	1.34	1.34	1.07	1.34	1.34	1.07	0.63	0.63	0.50	0.13	0.13	0.10
RLCO	0.11	0.11	0.35	0.11	0.11	0.35	1.15	1.15	3.69	0.76	0.76	2.44
ZNRG	-1.35	1.35	0.87	-1.35	1.35	0.87	0.40	0.40	0.26	-0.12	0.12	0.08
ARSN	-0.30	0.30	0.10	-0.30	0.30	0.10	0.86	0.86	0.28	0.39	0.39	0.13
AZ	-0.07	0.07	0.52	-0.07	0.07	0.52	0.08	0.08	0.64	0.03	0.03	0.20
KE	0.51	0.51	0.04	0.51	0.51	0.04	1.56	1.56	0.14	1.16	1.16	0.10
UH	-0.02	0.02	0.00	-0.02	0.02	0.00	3.58	3.58	0.26	2.35	2.35	0.17
TTRR	0.41	0.41	0.02	0.41	0.41	0.02	-3.00	3.00	0.11	-2.25	2.25	0.08
GNV	-0.78	0.78	0.78	-0.78	0.78	0.78	0.39	0.39	0.39	0.02	0.02	0.02
BNO	-0.04	0.04	0.01	-0.04	0.04	0.01	1.56	1.56	0.31	0.68	0.68	0.14
BGN	0.71	0.71	0.95	0.71	0.71	0.95	1.06	1.06	1.42	0.58	0.58	0.78
BGLV	3.17	3.17	0.08	3.17	3.17	0.08	0.08	0.08	0.00	0.47	0.47	0.01
ATCS	0.66	0.66	0.35	0.66	0.66	0.35	1.79	1.79	0.96	1.07	1.07	0.57
AZA	-1.03	1.03	0.68	-1.03	1.03	0.68	9.70	9.70	6.45	6.72	6.72	4.47
SILCF	0.75	0.75	0.80	0.75	0.75	0.80	0.66	0.66	0.71	0.24	0.24	0.25
JCOR	6.35	6.35	0.30	6.35	6.35	0.30	3.91	3.91	0.18	1.26	1.26	0.06
AMV	2.38	2.38	0.31	2.39	2.39	0.31	4.69	4.69	0.60	2.96	2.96	0.38
GSB	2.45	2.45	0.14	2.45	2.45	0.14	2.24	2.24	0.13	1.54	1.54	0.09
MLD	0.87	0.87	0.44	0.87	0.87	0.44	-0.15	0.15	0.07	-0.64	0.64	0.32
ZLC	0.20	0.20	0.02	0.20	0.20	0.02	1.60	1.60	0.13	1.09	1.09	0.09
NGCO	4.28	4.28	0.11	4.28	4.28	0.11	1.10	1.10	0.03	0.85	0.85	0.02
NCS	12.44	12.44	1.38	12.44	12.44	1.38	5.65	5.65	0.63	4.18	4.18	0.46
MEAN	0.51	1.04	0.41	0.51	1.05	0.41	1.21	1.74	2.47	0.59	1.15	1.71
MEDIAN	0.08	0.46	0.18	0.09	0.44	0.18	0.86	1.26	0.32	0.50	0.73	0.19
STD D	2.04	1.82	0.52	2.04	1.82	0.52	2.48	2.13	9.39	1.76	1.45	6.80
VAR'T	4.03	1.74	1.27	3.96	1.74	1.28	2.04	1.23	3.81	2.97	1.26	3.98

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THE THREE MILE ISLAND NUCLEAR ACCIDENT AND BUSINESS INTERRUPTION LOSSES: TWO CASE STUDIES

by

Robert C. Posatko*

I. Introduction

This paper presents a discussion of two cases of business interruption losses, which were a consequence of the nuclear accident in March of 1979 at the Three Mile Island (TMI) generating plant near Middletown, Pennsylvania. The TMI event spurred a variety of class action lawsuits on behalf of a number of groups allegedly affected by the incident, including personal injury and death claims, wage losses during work stoppage, evacuation costs, property value losses, and claims by firms suffering losses in business profits, and in some cases bankruptcy. Litigation resulted in a lump sum settlement between the utility company and the combined plaintiff groups, excluding the injury and death cases, which have been litigated separately.[1] Each of the affected groups then "competed" for recovery of losses and damages, and each individual plaintiff was evaluated according to criteria for the respective groups as established by the court and its representatives. The two cases discussed in this paper fall within the last group identified above, namely, claims of commercial-type damages.

In general, commercial loss plaintiffs were routinely awarded the equivalent of two weeks of gross profits on the assumption that this was the maximum duration and extent of losses by the average area firm. In many cases, however, particularly those of firms in or near Middletown itself, it was evident that damages were more complex and longer-term in nature. For a number of these, expert analyses and appraisals were conducted and reports presented in support of individual claimants' cases.

The two cases presented herein are that of a new car dealership, and a large, family-owned grocery store, both located within the immediate TMI geographic area (within 5 miles of the plant). In the car dealership case, the evidence suggested that losses occurred over a several year period, but appeared to have largely diminished by the time this analysis was completed in 1984. In the case of the grocery store, for which business volume is heavily linked to population levels in the Middletown area, annual earning losses were seen as extending beyond the date of the loss appraisal, at least to the year 1989, and conceivably beyond that year as the long-term appeal of this area as a place of residence appeared to have been affected.

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[1] A modicum of controversy still surrounds the question of public health effects from possible radiation emissions from the plant at the time of the incident. Approximately 2000 personal injury and death cases continue in litigation at present, some 19 years after the incident.

II. The Forecast Model

The methodology employed in calculating the economic losses of the two firms followed the general lines of analysis and estimation commonly used in business interruption cases of this type.[2] This involves, first, the forecasting of incremental revenue over the affected period (weeks, months or years, depending on circumstances) that would have been realized had the busniess interference not occurred. Available data on the sales determinants of the affected firms were examined for this purpose. Second, the associated incremental costs are evaluated and quantified. Since both cases involved retail sales activity, the cost of goods sold was the major variable cost, along with other selling expenses, including for example, commissions of the auto dealer's sales force. Third, consideration is given to any extraordinary costs (such as emergency advertising) that are motivated by the interruption itself. Finally, then, profit losses are the lost revenues, minus the avoided variable costs plus any added expenses prompted by the business interruption. For both of the cases presented, a fairly rich supply of historic operating data were available as a basis upon which to develop "but-for" estimates of revenues and costs, and useful external data sources were able to be developed for purposes of formulating the appraisals.

Business interruptions are commonly typed as "closed," "open," or "infinite" in terms of the duration of damages experienced.[3] The closed category of losses includes those for which the episode of damage has ended by the time the appraisal of losses is performed. In such cases, normal sales data from before and after the interruption are available for use in developing the estimate of losses in the intervening period. Open losses are those for which the firm's sales performance has not returned to normal by the time of the appraisal, but the firm continues to operate. The two cases discussed herein fall approximately within these first two categories of losses. By the time of final hearing of the auto dealership case in 1984, the interruption had largely diminished, though a full resumption of its "normal" sales path was not yet evident. The grocery sales case had the markings of an open, or continuing loss of business volume well beyond the appraisal and final settlement date.

Both cases illustrate the fact that analysts frequently are challenged as much by having to project the end of the loss episode as by having to project the size of losses. Furthermore, as noted by experienced practitioners in this field, the question of how far into the future to compute profit losses is often a legal issue as much as an economic one.[4] In the case of the TMI damages, the court initially imposed three to four year time limits (i.e. to 1981 or 1982) on damages, but subsequently did consider evidence on losses occurring up to the time of hearings which were largely completed by the end of 1984. Thus, while in a few select cases triggered by the TMI incident, actual damages might well have continued beyond 1984 (due to the longer-term harm to a business location) the court arbitrarily set a time limit on damage estimates it would consider.[5]

[2] For discussion of forecasting models and methodology frequently used in such business interruption cases, see Foster, Trout and Gaughan (1993) or Plummer and McGowin (1993).

[3] See Trout and Foster (1993), pp. 154-155.

[4] See Dunn (1989).

[5] See Case 2 below.

III. Case 1: Auto Sales, Inc.

A. Background

Case 1 involves a new and used car dealership ("Auto Sales, Inc.") in the community near TMI, which primarily sold a very popular foreign make, referred to in this paper as "ForCar," along with a make from one of the U.S. Big Three automakers, referred to as "Amercar." This dealership had been a successfully run enterprise for some 20 years as of the mid-1970's. In 1972 it began selling ForCar, augmenting its long-standing dealership in AmerCar. As shown in Table 1 and Figure 1, the firm's sales of ForCar had risen briskly from 1973 through 1978, roughly matching the sales path of its counterpart national average dealer. Unit sales by the average dealer nationwide, for the ForCar make of automobile, was derived from figures provided in the trade publication, Automotive News, Market Data Book. While not an exact replication of nationwide dealer growth rates over the 1973 to 1978 period, the plaintiff's sales record reasonably matches that of the national counterpart. Hence, the national average dealer growth rate was used as a benchmark in projecting the firm's sales, had the TMI incident not occurred.

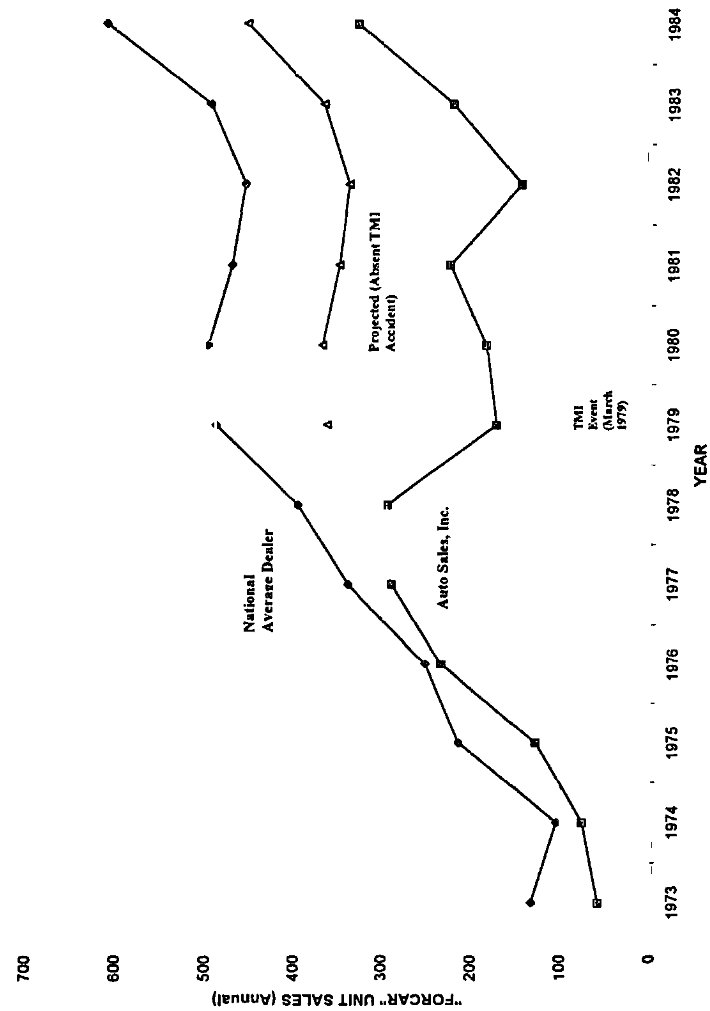
Following the incident, the plaintiff's unit sales for 1979 dropped sharply, and remained off the pace for the next 4 to 5 years. (It was management's strongly held view that throughout this several year period following the March 1979 incident, sales and profits were hurt because many potential customers from outside the immediate vicinity were reluctant to buy from this dealership and return for subsequent service visits due to imagined negative health consequences in the area near the plant.)

Table 1
Auto Sales, Inc.
ForCar Unit Sales

Year	National Average Dealer*	(Annual Growth Rate)	Auto Sales, Inc. (Actual)	Auto Sales, Inc. (Projected-- Absent TMI Incident)
1973	132		57	
1974	104		75	
1975	214		127	
1976	252		234	
1977	339		290	
1978	396		295	
1979	488	23.23%	172	363
1980	496	1.60%	183	368
1981	470	(5.24%)	224	349
1982	455	(3.19%)	144	338
1983	494	8.57%	221	367
1984	610	23.48%	328	453

*** Source:** Automotive News, Market Data Book, (annual, 1973-1984) published by Crain Communications, Inc.

FIGURE 1
UNIT SALES: NATIONAL AVERAGE DEALER AND AUTO SALES, INC.



Sales of AmerCar, a product of one of the U.S. Big Three, which had been very stable prior to 1979, dropped sharply in the accident year and appeared to have been moderately dampened over the next 4 years as well. AmerCar sales appear in Table 2.

B. Estimation of Lost Sales and Profits -- ForCar

The estimate developed for the firm's losses in new car sales of both ForCar and AmerCar vehicles was based on the sales performance of dealers nationwide selling these two makes of automobiles. This benchmark, or "yardstick", represented by national average dealer unit sales, had applicability in this case as other market factors, including the number of competing dealers in the area, were constant during the affected period.[6] Use of this multi-year benchmark was particularly valuable in the present case since it appeared that the impact on the company extended for several years beyond 1979 due to the apparent, lingering tendency on the part of many residents of the greater Harrisburg region to avoid the Middletown area if they could do so. The period of impact thus included the 1981-1982 recession period, and it was assumed that this dealership would have suffered the same degree and timing of lost sales during that recession as the counterpart national average dealer. In recent decades, the state of Pennsylvania had experienced economic fluctuations of quite similar duration and amplitude as those of the nation as a whole. During the most significant recession period prior to the TMI incident, the 1973-1975 contraction, the Pennsylvania unemployment rate tracked very closely with that of the U.S. economy overall.[7] Within the more proximate Harrisburg region, recession period data, including the 1981-1982 episode, indicate a strong correlation of the region's unemployment rate with that of the national economy.[8]

[6] For a discussion of other yardstick or benchmark indicators, see Foster, Trout and Gaughan, (1993), p. 189.

[7] In the U.S. as a whole, the unemployment rate rose from some 4.8% in 1973 to 5.6% in 1974 and peaked at 8.6% in early 1975; correspondingly, in Pennsylvania the unemployment rate was 4.8% in 1973, 5.3% in 1974 and 8.4% in 1975 (See Choices for Pennsylvanians, December 1980, p.16)

[8] See: Regional Economic Update, "Unemployment Rate Characteristics of Metropolitan Area Economies," (Autumn 1995).

Table 2
Auto Sales, Inc.

Year	National Average Dealer* (Annual Growth Rate)		Auto Sales, Inc. (Actual)	(Projected - Absent TMI Incident)
1974	158		36	
1975	192		79	
1976	272		104	
1977	294		102	
1978	302		114	
1979	286	(5.30%)	72	108
1980	251	(12.24%)	60	95
1981	263	4.78%	88	100
1982	254	(3.42%)	60	96
1982	320	25.98%	145	121

***Source:** Automotive News, Market Data Book (annual, 1974-1983), published by Crain Communications, Inc.

The utilization of the unit sales record of the national average dealership was straight- forward. The annual rates of change of the average dealer from 1978 onward were applied to the plaintiff's pre-TMI (1978) unit sales, generating projections of annual sales of the ForCar make for the years 1979 through 1984. Making direct use in this way of the actual year-to-year volume of the average dealer to estimate "but-for" sales of the damaged firm seemed especially appropriate for two reasons: 1) as noted above, it preserved the effects of the 1981-1982 recession on projected sales for the plaintiff; and, 2) it captured the particular timing of the growing appeal and profitability of this increasingly popular make of automobile. Figure 1 thus includes the projected sales levels of the car over the 1979 to 1984 period, the time-frame affected by the nuclear incident.

Table 3 summarizes the loss calculations for this make of car for the period of 1979 through 1983.[9] Subtracting actual units sold from projected levels yields lost sales, in units. Multiplying by the average price of cars actually sold each year, and by the average gross profit per unit from actual sales, produced estimates of lost sales revenue and lost gross profits, respectively. Gross profit per car earned on the ForCar make varied substantially over the 1979 to 1983 period from 13 percent to 19 percent of sales, reflecting changes in features provided as standard equipment, sticker prices, and the mix of models sold.

A detailed examination of the firm's accounting records indicated that the likely added variable costs that would accompany such additional sales ranged from about 5

[9] As noted above, the court established a more-or-less fixed end point on business loss recovery, which in the present case meant to the end of 1983. As suggested by Figure 1, actual losses for this firm appear to have continued into 1984, and may have extended beyond.

percent to 6 percent of gross sales, the bulk of which was commissions of 4percent, and advertising of about ½ percent. Netting out these additional sales costs (using 5.5 percent of gross sales) from the unrealized gross profit yielded the losses in net profit on the ForCar make over the five-year period.

C. Estimation of Lost Sales and Profits -- AmerCar, Used Cars, and Parts and Service

An estimation of the firm's losses on the American make of auto it sold was similarly developed for the 1979 to 1983 period. Based also on a comparison with the national average dealer sales performance, annual sales of this make declined following the TMI incident, but were less dramatically reduced than were sales of the foreign-made vehicle. This differential effect was attributed to the fact that the clientele of the American make of car consisted more heavily of local residents of the vicinity of TMI, as opposed to customers from outside this vicinity who tended to avoid doing business in the area of the TMI plant.[10]

Table 3
Auto Sales, Inc.
ForCar: Lost Sales, Revenues, and Net Profit

	For Car Units			Lost	Lost	Lost
	Actual	Projected	Lost	Gross	Gross	Net
Year	Sales	Sales	Sales	Revenue	Profit	Profit
1979	172	363	191	$1,045,077	$161,096	$103,617
1980	183	368	185	1,369,837	260,269	184,928
1981	224	349	125	1,287,177	197,244	126,449
1982	144	337	194	1,463,662	190,885	110,384
1983	221	367	146	1,241,427	170,075	101,797

Losses in business volume and profit reductions were also evaluated for used car as well as service and parts sales by the dealership. The historical pattern for all three of these components of dealership business indicated a very strong linkage to new car sales. In the five years prior to 1979 the ratio of used car sales to total new car sales averaged 28.7 percent and in three of those five years, the percentages were consistently in the 28.0 percent

[10] Sales records of the firm indicated that for the three-year period prior to the accident approximately twice as many purchasers of ForCar automobiles resided outside of Middletown and the three contiguous boroughs to that of the sales location as did purchasers of the American make of automobile.

to 28.5 percent range. Using the mid-point value in this range, or 28.25 percent, estimates of the lost used car sales volume (absent TMI) were generated from the lost ForCar and AmerCar sales, developed above. Netting out variable costs associated with used car activity yielded estimates of annual losses in net profit from this component of operations. Similarly, parts and service department losses were computed on the basis of their having averaged some 5.1 percent of total company sales in the 5 years before the 1979 incident. With variable costs in these two departments exhibiting a steady relationship with sales, losses in net profit were then computed in straight-forward fashion.

IV. Case 2: Frank's Foods, Inc.

A. Background

Frank's Foods, a successful, family-owned, retail grocery business, is located in the main shopping district of Middletown, Pennsylvania. By the mid 1970's with a size of some 30,000 square feet, the firm had established itself as offering the full range of food and related items on a par with stores of the major chains. Annual sales revenue had been growing continuously, at an average of some 12.5 percent per year, in the four years prior to the nuclear accident. In the immediate period of the TMI incident and brief closing of the store, damages amounted to some three days of net profit losses, and spoilage of inventory of perishable and other items. These very short-run losses were estimated by the firm's ownership itself in collaboration with the firm's accountant, and totaled to some $20,300.

This author's involvement in the case was enlisted to assess the longer-term impact on the grocery firm. As a result of the nuclear incident, the image of Middletown, Pennsylvania as a place to live, had certainly been adversely affected. A study sponsored by the Nuclear Regulatory Commission in early 1980 showed that 30 percent of the residents within a 5 mile radius of the plant had considered moving because of the accident. Though out-migration of this scale did not occur, the survey was indicative of a strong change in sentiment on the part of local residents. Also, those relocating within the Harrisburg region as well as those newly-moving to the region were now more likely to avoid the Middletown area as a place of residence. General information from real estate firms with offices serving Middletown indicated a significant decline was experienced in referrals from other areas, following the accident.

Data on residential building permits indicated a reluctance to place new homes (including mobile homes) in the Middletown area. Figures were collected on permits issued for many of the major municipalities in Dauphin County (including Middletown Borough), and for the county as a whole. Annual averages from these data were calculated for each municipality for the pre-accident period of 1970-78, and for the three available post-accident years of 1979-81. While every municipality experienced a slowdown in construction in line with national trends, the decline by Middletown Borough, from an average of 123 permits per year prior to 1979 to 11 per year from 1979 on, was by far the most dramatic. (See Appendix A.)

In sum, there were numerous indications of resistance to the immediate TMI area, including Middletown, as a place of residence. This municipality is the principal source of customers for Frank's Foods, Inc. To the extent that the population in the borough was smaller over time than it would have been had the accident at TMI not occurred, the firm experienced longer-term economic losses due to the incident. In 1982, at the time of this evaluation, all indications were that business interruption losses would thus continue beyond the time of appraisal, constituting an "open" type of loss, as defined above. Population data for Middletown and for comparison areas, which are presented below, suggested long-term losses of this nature.

B. Population Trends for Middletown and Comparison Areas: 1970-78 and 1979-82

Table 4 provides population data for Middletown Borough for the period 1970 through 1982. These data, as well as those for two other comparable areas, were drawn from annual springtime census tabulations conducted by school districts throughout Pennsylvania. For the period of 1970 to 1978 (pre-TMI), Middletown experienced growth in population averaging 1.43 percent annually. After 1978 the average annual rate of change in population fell to 0.32 percent. In the same table, comparable population data are shown for the Lower Dauphin and Central Dauphin School Districts. All three of these districts serve the function of being bedroom communities for individuals employed in the greater Harrisburg area, and their per capita income levels are similar. For the 1970-78 period, population growth in the Central Dauphin district averaged 1.83 percent per year, and it averaged 0.96 percent per year between 1978 and 1981 (1981 was the most recent data year available for the Central and Lower Dauphin districts at the time of this report). For Lower Dauphin, the data, which were available only from 1973 on, indicated 2.71 percent average annual growth from 1973 to 1978, and 1.45 percent average growth from 1978 to 1981. Though both districts experienced a slowing of population growth, neither exhibited an absolute decline and subsequent near-leveling as did Middletown. While these two otherwise similar districts after 1979 had population growth averaging some 53 percent of pre-TMI levels, Middletown's growth rate after 1979 was initially negative, then positive but minuscule after 1980. Therefore, it was reasonable to estimate that were it not for the TMI accident, Middletown's growth after 1978 would have been about the same 53 percent of its pre-TMI rate. This implies an average population growth of approximately 0.75 percent per year.

Table 4
Frank's Foods, Inc.
Population Levels and Rates of Change for
Middletown, Lower Dauphin School District, and
Central Dauphin School District: 1970-1982

Year	Middletown Borough Population	Lower Dauphin Population	Center Dauphin Population
1970	8,730		57,894
1971	8,965		58,584
1972	9,233		59,590
1973	9,452	16,827	61,640
1974	9,755	17,282	62,868
1975	9,920	18,131	64,794
1976	10,060	18,560	65,738
1977	10,103	18,793	66,649
1978	9,765	19,222	66,931
1979	9,408	19,482	67,318
1980	9,760	19,768	68,579
1981	9,803	20,073	68,878
1982	9,879		
Average annual change prior to 1979 (per above)	1.43%	2.70%	1.83%
Average annual change subsequent to 1978 (per Above)	0.32%	1.45%	0.96%

Sources: Lower Dauphin, Central Dauphin and Middletown School Districts.

Using this 0.75 percent per year estimate of the growth rate of Middletown, absent the accident, a series of projected population levels from 1980 through 1989 were generated. These are shown in Table 5. The second column reports actual population count through 1982, and for 1983 through 1989, the probable population levels based on growth rates after the accident (0.32 percent per year). The third column then represents the estimated loss in population in Middletown due to the accident, which served as the basis for computing the sales loss to Frank's Foods. At the time of this analysis, officials at TMI estimated that it would be 1988 or 1989 before the work of clean-up and repair of the damaged reactor was completed. Thus, the appeal of Middletown as a residential location was assumed to be impaired at least until that point in time, and therefore sales losses were estimated through the year 1989.

C. Sales and Profit Losses to Frank's Foods: 1980-1989

As shown in Appendix B, a computation was made of the annual number of store customers per person living in Middletown. For the years 1978 through 1981 this ratio remained relatively constant, and averaged 66.46. Thus, this figure was utilized in projecting the magnitude of customer losses over the period of 1982 to 1989. For the years 1980 and 1981 slightly different values, reflecting the actual ratios for these years, were used. Certainly not all customers of Frank's Foods were Middletown residents. Based on occasional surveys and personal knowledge of their clientele, ownership estimated that 40 percent of the store's customers came from outside of the borough. Hence, the above ratio was adjusted (multiplied) by a factor of 60 percent. The resulting number (66.46 x .6 = 39.87) provides the number of customer visits by Middletown residents per person in the borough of Middletown. This suggests that for a family of 4, approximately 160 visits were made to the store per year, or about 1 visit every 2.3 days. This is a reasonable frequency in view of Frank's proximity to the population and the large amount of "walk-in" trade the store had experienced.

The next step was to place a dollar value on the sales which would have been realized from the additional population. Average purchases per customer for 1980 and 1981 were $13.18 and $12.97, respectively. For the years 1982 through 1989 a rounded $13 per customer transaction was used in forming the loss computation. Thus, the yearly impacts of the TMI accident on gross sales at Frank's beginning with the first full year after the accident (1980), were calculated as shown in Table 6.

Accounting records of the firm indicated that gross profit as a percentage of gross sales had averaged 21.14 percent over the period of 1978 to 1982, and presumably was not affected by the TMI incident. It was assumed that this same gross profit percentage would have been realized on the lost sales due to TMI. In turn, virtually all of the added gross profit would have accrued as net(pre-tax) profit to the firm. This is the case because such modest additional volumes of business (representing about 1 percent of sales in 1980) would not have triggered significant added costs; the firm's labor, utility, advertising, and other operating costs would remain essentially unchanged. Thus, the resulting loss of net profit to Frank's Foods was estimated at 20 percent of the loss in yearly gross sales. For the two years prior to completion of this report in late 1982 (1980 and 1981) the lost profit was $33,883. For the years 1982 through 1989 losses in net profit were effectively computed in present value terms as of 1982 (see Table 6 note) and sum to $281,442.

Table 5

Frank's Foods, Inc.
Projections of Middletown Population

Year	Projected Population of Middletown - Absent TMI Accident	Population of Middletown - as Affected by TMI Accident (actual population)*	Population Loss Due to TMI Accident
1979	9,838	9,408	
1980	9,912	9,760	152
1981	9,986	9,803	183
1982	10,061	9,879	182
1983	10,137	9,910 (est.)	227
1984	10,213	9,942 (est.)	271
1985	10,289	9,974 (est.)	315
1986	10,367	10,006 (est.)	361
1987	10,444	10,038 (est.)	406
1988	10,532	10,070 (est.)	453
1989	10,602	10,102 (est.)	500

* Population figures for 1983 through 1989 are based on a 0.32 percent per year growth rate, the average growth rate in the post accident period through the year 1982, the last year for which data were available at the time of this analysis.

Table 6
Frank's Foods, Inc.
Losses in Population and Sales

Year	Middletown Population Differential Due to TMI		# of Middletown Customer Visits Per Year Per Middletown Resident		Average Expenditure Per Customer	Loss in Gross Sales*
1980	-152	x	38.30	x	$13.18	$76,729
1981	-183	x	39.05	x	12.97	92,685
1982	-182	x	39.87	x	13.00	94,332
1983	-227	x	39.87	x	13.00	117,656
1984	-271	x	39.87	x	13.00	140,462
1985	-315	x	39.87	x	13.00	163,268
1986	-361	x	39.87	x	13.00	187,110
1987	-406	x	39.87	x	13.00	210,434
1988	-453	x	39.87	x	13.00	243,794
1989	-500	x	39.87	x	13.00	259,155
					Total Loss in Gross Sales	$ 1,576,625

* Future dollar values in this column (for the years 1983 through 1989) were neither adjusted upward for the inflation in product prices which would occur, nor discounted to factor out the time value of money (i.e., the interest rate). It was assumed these two factors would approximately offset each other.

IV. Conclusions

The conventional and widely-used business interruption model served well as a general framework for determining losses in the two cases described above. In each case, however, relatively unique data sources were drawn upon for estimating the most difficult component of such business loss appraisals, namely, the "but-for" estimate of revenue. In the auto dealership case, the yardstick measure derived from average national dealer sales proved to be a credible predictor for this purpose. Such trade association data may constitute a fruitful source of information for forensic economists undertaking similar forecasts in other retail, wholesale or production activities. The author also notes that the use of a large visual exhibit of the actual vs. projected sales, shown in Figure 1, served quite effectively in the courtroom presentation of this case.

Damages to the grocery store (beyond the immediate closing and spoilage losses) were seen as longer-term and "open," in effect constituting a delayed and cumulating consequence of the accident, as reduced population growth affected sales. Projection of lost sales in this case was by nature more speculative than that of the first case. Both the quality and time-length of school census population data used for estimating lost revenue were of some concern. Ultimately, the estimate of a 500 person, or 5 percent, dampening in the population of Middletown by a point ten years after the accident seemed within the bounds of reasonableness.

A related issue was whether there might be continuing, though perhaps diminishing, losses which extended beyond the damages described in this paper, generating some loss in going concern value for the subject firms. In the TMI litigation, the court in most cases ruled out consideration of losses beyond 1983, and thus effectively denied compensation for loss in value resulting from future earnings losses. Aside from the calculation of grocery store losses over the 1982 to 1989 period, no loss in value from future profit reductions was developed in this author's work. If, prior to settlement, either of the firms had been sold, or was under sales negotiations, loss in value would have been raised as an issue and given consideration by the court. Speculation about more permanent damage to firms in the Middletown area (and loss in value) presumably would have hinged on the absence of an eventual "decay" in the health concerns and negative image of the Middletown area as a retail location. Both enterprises, of course, had the opportunity in the long run to mitigate any expected longer-term losses through relocation, or the opening of satellite locations away from the original places of business.

Appendix A
Frank's Foods, Inc.
Building Permits in Major Municipalities of Dauphin County
and Dauphin County: 1970-1978 and 1979-1981

	Annual Average Number of Permits Issues 1970-1989	Annual Average Number of Permits Issues 1979-1981	Ratio of 1979-1981 Average to 1970-1978 Average
Derry Township	157.3	64.3	0.41
Harrisburg City	69.0	41.0	0.59
Hummelstown Boro	15.3	10.3	0.67
Lower Paxton Township	644.2	278.3	0.43
Lower Swatara Township	59.2	39.3	0.66
Londonderry Township	26.4	16.0	0.61
South Hanover Township	53.5	35.6	0.67
Steelton Boro	21.8	9.6	0.44
Swatara Township	202.2	90.3	0.45
West Hanover Township	52.2	33.3	0.64
Dauphin County	1837.2	883.3	0.48
Middletown Borough	123.6	11.3	0.09

Source: Tri-County Regional Planning Commission, Harrisburg, Pennsylvania

Appendix B
Frank's Foods, Inc.

Projections of Customer Visits as
Related to the Population of Middletown

Year	Number of Customer Visits	Middletown Population	Ratio of Number of Customer Visits to Population of Middletown
1978	659,202	9,765	67.50
1979	652,950	9,408	69.40
1980	622,987	9,760	63.83
1981	638,105	9,803	65.09
Average for 1978-1981			66.46

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WORK LIFE EXPECTANCIES OF NONSMOKERS, LIGHT SMOKERS, AND HEAVY SMOKERS

by

James Ciecka, Thomas Donley, Seth Epstein and Jerry Goldman*

I. Introduction

In a recent paper, Ciecka and Goldman (1995) developed estimates of the work life expectancies of smokers *vis-a-vis* nonsmokers. Such a distinction is useful for a variety of reasons, not the least of which is to aid in the valuation of wage and pension losses in connection with litigation arising from wrongful death and personal injury lawsuits. However, despite the fact that few issues have so captured the attention of society as have smoking and smoking-related illnesses, precious little published data exist on the mortality rates of smokers compared with nonsmokers. Consequently, Ciecka and Goldman relied on mortality rate tables developed by a task force of The Society of Actuaries and published as a *Transactions of the Society of Actuaries Committee Report* (1982 and 1987) which broadly classifies individuals into two categories: smoker or nonsmoker.[1] This report blended mortality experiences of five insurance companies for the years 1973-78. In addition to the somewhat dated nature of the data, the broad smoker/nonsmoker classification, and the fact that only insured lives were included in the Society of Actuaries report, mortality estimates may have been affected by definitional differences among insurance companies (*e.g.*, pipe and cigar smokers could qualify as nonsmokers in two companies, and three companies included substantial numbers of nonsmokers in their smoking group).

New work by Hummer, Nam , and Rogers (forthcoming) greatly adds to the literature by providing more current smoker mortality data and data on separate categories of smoking behavior. The purpose of this paper is to incorporate these finer distinctions into the original Ciecka-Goldman work to yield age-specific gender-based estimates of work life expectancies for light and heavy smokers and to incorporate the most current transition probabilities into these estimates (Ciecka, Donley, and Goldman, 1995 and 1996). Our results indicate meaningful differences between light and heavy smokers that could not be captured previously.

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[1] Since the Society of Actuaries report is based on insurance company records, there may be some sample selection bias related to factors such as income and wealth if higher income and wealthier individuals more commonly purchase life insurance than lower income and less wealthy people. Individuals with higher incomes and greater wealth may be healthier and receive better medical care and thus experience lower overall mortality rates than their lower income and less wealthy counterparts given the same amount of smoking. In addition, the underwriting process itself may result in the systematic exclusion of less healthy individuals given the same amount of smoking.

II. Summary of Existing Mortality Data and Life Expectancies by Various Smoking Categories and New Work Life Expectancies by Various Smoking Categories

Hummer, Nam, and Rogers present three distinct categories of smoking behavior.[2] First, Never Smoked, are those who have smoked fewer than 100 cigarettes in their lives; second, Light Smokers, are those who have smoked at least 100 cigarettes, continue to smoke, but smoke fewer than 25 cigarettes per day on average; finally, the third category, Heavy Smokers, differs from Light Smokers in that they average 25 or more cigarettes per day. Based on the data available these authors were forced to choose between a cutoff point of 15 or 25 cigarettes per day. The choice of 25 was made to ensure that Heavy Smokers consumed a minimum of one pack (20 cigarettes) per day.[3]

Tables 1 and 2 contain mortality rates for men and women falling into the three categories of smoking behavior broken into ten-year age intervals in Columns (2)-(4). Column (5) contains rates without regard to smoking; Column (6) is provided for comparison, showing mortality rates for the entire population compiled from *Vital Statistics of the United Sates, 1986* (1988). The great similarity in the two final columns lends substantial credibility to the work of Hummer, Nam, and Rogers. The entries in parentheses in Tables 1 and 2 are relative mortality ratios, using the Total HNR (in Column 5) in the denominator of each ratio. [See Slesnick and Thornton (1994) and Thornton and Slesnick (1997) for a discussion and estimates of life expectancies under different relative mortality ratio assumptions.]

Closely related to the topic of mortality rates are life expectancies. In a study by Rogers and Powell-Griner (1991), life expectancies for men and women are calculated.

[2] Two additional categories, Long-term Former, and Recent Quit were excluded from this paper. The first of these did not differ significantly from those who never smoked, and the second category contained many individuals who had quit because they were already suffering from serious health problems. Long-term Former smokers are those smokers who quit more than one year ago, and Recent Quits are those who stopped smoking within the last year.

Hummer, Nam, and Rogers combined three data sets in their study: the *1986 National Mortality Followback Survey* (NMFS), the *1985 National Health Interview Survey*, Health Promotion and Disease Prevention Supplement (NHIS-HPDP), and the *1987 National Health Interview Survey*, Cancer Risk Factor Supplement, Epidemiology Study (NHIS-ES). The 1986 NMFS provides data on deaths and the 1985 and 1987 NHIS studies contain data on the general population at risk. The NMFS is a nationally representative sample of one percent of the population who died in 1986 and who were age 25 or older. The NHIS samples contain individuals age 18 and older, and the samples for 1985 and 1987 were combined to yield approximately 44,000 cases after deleting data for those individuals younger than age 25.

[3] It may be worth noting that 25 cigarettes is also the cutoff point used by the manufacturer of Nicorette gum; the gum comes in two and four milligram dosages, and the company recommends that the lower dose be used by smokers of under 25 cigarettes per day, and the higher dosage be used by those smoking 25 or more per day.

Tables 3 - 6 deal with life expectancies for the three types of smoking status for white men and women.[4] The significant result is the large difference between light and heavy smokers; this result carries through to work life expectancies but in a far less dramatic manner. Although both light and heavy smokers (male and female) have lower life expectancies than their non-smoking counterparts at every age bracket, male light smokers, beginning at age 55 actually have a higher than average life expectancy, which we hypothesize is due to the possibility that heavy smokers pull the average down so dramatically. For women, the influence of heavy smokers on the average appears immediately, and light smokers have a higher than average life expectancy for all age groups.

As can be seen in Tables 3 - 6, smoking has a seriously detrimental effect on life expectancy; it stands to reason that smoking may also reduce work life expectancy. Tables 7 and 8 contain, for men and women, the ratios of work life expectancies for various smoking categories to work life expectancies without regard to smoking at various exact ages; further, a breakdown between those active and inactive in the labor force is provided. (The Appendix contains a more detailed explanation of the methods used to calculate Tables 7 and 8.) Tables 9 and 10 contain work life expectancies at the same exact ages used in Tables 7 and 8.[5]

As would be expected, among both men and women, at all age groups smoking reduces work life expectancy relative to having never smoked. However, the substantial difference between light and heavy smokers again appears, and this effect is especially noticeable among women. Women who are light smokers show little difference in work life expectancy compared to those who never smoked. In contrast, both men and women who smoke heavily show a more marked decrease in work life expectancy in percentage terms, and this effect is more pronounced with age until age 65. For heavy smokers, inactive men at 55 have a 15 percent reduction in work life expectancy and women show 14 percent drop relative to the population without regard to smoking status. Being active in the labor force is associated with lesser effects of smoking (for both men and women), as active participants have an equal or higher work life expectancies than their inactive counterparts.

Differences in work life expectancy across the three smoking categories for active men and women are depicted in Figures 1 and 2. In each figure, the top curve is the difference between work life for those who never smoked and work life without regard to smoking status, the middle curve is the difference for light smokers relative work life without regard to smoking status, and the bottom curve refers to work life expectancies of heavy smokers less work life without regard to smoking. Because these differences are not in percentage terms, the largest differences are for younger individuals. But the striking impact of heavy over light smoking remains, particularly for women. Work life expectancy for light-smoking women lies virtually across the horizontal axis, indicating nearly identical work lives to the entire female population without regard to smoking status.

[4] The implied mortality rates in Tables 3 and 4 are smaller than those in Tables 1 and 2 even though the Hummer, Nam, and Rogers study utilizes the same data sets as the Rogers and Powell-Griner study because the latter is restricted to the white population.

[5] Tables 1-2 and 7-10 refer to all men and women without regard to race; in Tables 3 - 6, race is restricted to only white men and women. Tables 3-10 are for the exact ages indicated, while Tables 1 and 2 refer to the indicated age groups.

Conclusions

The Ciecka-Goldman work life expectancies for male smokers are approximately 4 percent-8 percent lower than the work life expectancies for the entire male population without regard to smoking status for ages 20-55. Columns (5) - (7) of Table 7 show at most a 2 percent reduction in work life expectancies of Light smokers relative to the entire population for ages 20-55. However, All and Active Heavy smokers (Columns 8 and 9) exhibit 5 percent-10 percent shorter work lives over the same age range. Heavy inactive male smokers (Column 10) show a 5 percent-15 percent decline in their work lives.

The Ciecka-Goldman work life expectancies for women smokers differ very little from work life expectancies for the entire female population without regard to smoking status. From ages 20 to 55, the work life expectancies for women smokers were only approximately 2 percent below the work life expectancies for the entire female population for active, inactive and all women. Similarly, as can be seen from Columns (5) - (7) of Table 8, there is virtually no difference between work life expectancies of Light smokers and the entire female population. However, All and Active Heavy women smokers (Columns 8 and 9) exhibit 4 percent-8 percent shorter work lives between ages 20 and 55. Heavy inactive women smokers (Column 10) experience a 4 percent-14 percent decline in their work lives in this age interval. Heavy smoking has a large impact on life expectancies but a smaller impact on work life expectancies. In Tables 5 and 6, life expectancies of Heavy Smokers are between approximately 20 percent and 35 percent lower than for the entire population between ages 25 and 55. In contrast, work life expectancies are only 4 percent to 10 percent lower. This is understandable because smoking causes long-term health problems and monotonically increasing mortality impacts with age. Since the largest mortality effects occur at mid-life and beyond (including years after which many people have left the labor force), work life expectancies are less affected than life expectancies. However, our work life expectancy estimates do not capture the effects of smoking induced morbidity. People may leave the labor force temporarily or permanently because of smoking induced illnesses. Although such people are inactive, our study excludes them until they ultimately die. To the extent that smoking leads to early labor force withdrawals prior to death, we have underestimated the work life effects of smoking.

Table 1
Mortality Rates per 100,000 and Relative Mortality Ratios for
Various Smoking Categories for Men by Age Groups in 1986

Ages (1)	Never Smoked (2)	Light Smoker (3)	Heavy Smoker (4)	Total HNR (5)	Total Vital Statistics (6)
25-34	138 1 (.71)	284 3 (1 46)	353 5 (1 81)	195 3	193 2
35-44	214 9 (.75)	430 0 (1 50)	462 6 (1 62)	286 2	295 5
45-54	357.3 (.56)	835 4 (1 31)	1,029 6 (1 62)	635 6	658 4
55-64	1,220.2 (74)	1,592 2 (96)	3,199 4 (1 94)	1,651 6	1,651 2
65-74	2,861 4 (82)	3,808 6 (1.09)	8,574 2 (2 46)	3,485 3	3,781 8

Source: Columns 2-5 Hummer, Nam and Rogers (HNR), Column 6 Calculated from *Vital Statistics of the United States, 1986* -- average mortality probabilities within each age group.

Table 2
Mortality Rates per 100,000 and Relative Mortality Ratios for Various Smoking Categories for Women by Age Groups in 1986

Ages	Never Smoked	Light Smoker	Heavy Smoker	Total HNR	Total Vital Statistics
(1)	(2)	(3)	(4)	(5)	(6)
25-34	63 3 (90)	79.2 (1 13)	132.0 (1 88)	70.1	71 9
35-44	110.5 (.80)	150.5 (1.09)	244 8 (1.78)	137 5	144.8
45-54	226 8 (.64)	342 9 (.97)	951.6 (2.70)	352 4	367.5
55-64	571.7 (66)	841.5 (.97)	2,138 8 (2.46)	870 3	905 8
65-74	1,556 7 (.79)	2,096 5 (1 06)	7,469.5 (3 78)	1,976.4	2,126.3

Source: Columns 2-5 Hummer, Nam, and Rogers (HNR), Column 6 Calculated from *Vital Statistics of The United States, 1986* -- average mortality probabilities within each age group

Table 3
Life Expectancies in Years by Various Smoking Categories for White Men by Age Groups in 1986

Age	Never Smoked	Light Smoker	Heavy Smoker	Total RPG	Total Vital Statistics
(1)	(2)	(3)	(4)	(5)	(6)
25	52.7	48.7	40.8	49.4	48.8
30	48.1	44.3	36.4	44.8	44.2
35	43.3	39.8	31.0	40.2	39.5
40	38.7	35.3	27.5	35.6	34.9
45	34.1	30.9	23.2	31.1	30.4
50	29.4	26.5	19.0	26.6	26.1
55	25.0	23.0	15.0	22.6	21.9
60	21.0	19.1	11.9	18.9	18.2
65	17.3	15.8	8.6	15.6	14.8
70	13.7	13.0	6.0	12.5	11.7

Source: Columns (2) - (5) Rogers and Powell-Griner (RPG)
Column (6) - *Vital Statistics of the United States, 1986*

Table 4
Life Expectancies in Years by Various Smoking Categories for White Women by Age Groups in 1986

Age	Never Smoked	Light Smoker	Heavy Smoker	Total RPG	Total Vital Statistics
(1)	(2)	(3)	(4)	(5)	(6)
25	60.6	57.5	44.2	55.7	55.0
30	55.7	52.7	39.4	50.8	50.1
35	50.9	47.8	34.6	46.0	45.3
40	46.1	43.0	29.8	41.2	40.5
45	41.3	38.3	25.2	36.5	35.8
50	36.6	33.6	20.8	31.8	31.2
55	32.0	29.2	17.2	27.5	26.8
60	27.5	25.2	9.9	23.3	22.6
65	23.4	21.2	7.0	19.4	18.7
70	19.5	17.8	6.6	15.8	15.1

Source: Columns 2-5, Rogers and Powell-Griner (RPG)
Column 6, *Vital Statistics of the United States, 1986*

Table 5
Ratios of Life Expectancies by Various Smoking Categories for White Men by Age Groups in 1986 to Total Life Expectancy

Age (1)	Never Smoked divided by Total RPG	Light Smoker divided by Total RPG	Heavy Smoker divided by Total RPG
	(2)	(3)	(4)
25	1.067	.986	.826
30	1.074	.989	.813
35	1.077	.990	.771
40	1.087	.992	.772
45	1.096	.994	.746
50	1.105	.996	.714
55	1.106	1.018	.664
60	1.111	1.011	.630
65	1.109	1.013	.551
70	1.096	1.040	.480

Source: Columns 2-4 calculated from Table 3.

Table 6
Ratios of Life Expectancies by Various Smoking Categories
for White Women by Age Groups in 1986 to Total Life Expectancy

Age (1)	Never Smoked divided by Total RPG (2)	Light Smoker divided by Total RPG (3)	Heavy Smoker divided by Total RPG (4)
25	1.088	1.032	.794
30	1.096	1.037	.776
35	1.107	1.039	.752
40	1.119	1.044	.723
45	1.132	1.049	.690
50	1.151	1.057	.654
55	1.164	1.062	.625
60	1.180	1.082	.425
65	1.206	1.093	.361
70	1.234	1.127	.418

Source: Columns 2-4 calculated from Table 4.

Table 7
Ratios of Work Life Expectancies by Various Smoking Categories to Work Life
Expectancies Without Regard to Smoking Status, for Men, by Exact Age

Never Smoked WLE divided by Total WLE				Light Smoker WLE divided by Total WLE			Heavy Smoker WLE divided by Total WLE		
(1) Age	(2) All	(3) Active	(4) Inactive	(5) All	(6) Active	(7) Inactive	(8) All	(9) Active	(10) Inactive
20	1 02	1.02	1 02	0 98	0 98	0 98	0 95	0 95	0 95
25	1 02	1 02	1.02	0 98	0 98	0 98	0 95	0 95	0 94
30	1 02	1 02	1 03	0 98	0 98	0 98	0 95	0 95	0 94
35	1 02	1.03	1 03	0 98	0 98	0 98	0 95	0 94	0 94
40	1 03	1 03	1 03	0 99	0 98	0 98	0 94	0 94	0 93
45	1 03	1 03	1 04	0 99	0 99	0.99	0 93	0 93	0.91
50	1 02	1.03	1 03	1 00	1 00	1.00	0 93	0 92	0 89
55	1 02	1 03	1 04	1 00	1 00	1 00	0 91	0 90	0 85
60	1 02	1 02	1 04	1 00	1 00	1 00	0 91	0 90	0 81
65	1 01	1 02	1 04	0 98	1 01	1 00	0 88	0 86	0 73
70	1 01	1 01	1 03	0 99	0 99	0 98	0 94	0 90	0 77

Source: See Appendix

Table 8
Ratios of Work Life Expectancies by Various Smoking Categories to Work Life Expectancies Without Regard to Smoking Status, for Women, by Exact Age

Never Smoked WLE divided by Total WLE				Light Smoker WLE divided by Total WLE			Heavy Smoker WLE divided by Total WLE		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age	All	Active	Inactive	All	Active	Inactive	All	Active	Inactive
20	1 01	1 01	1.01	1 00	1 00	1.00	0 96	0.96	0 96
25	1 01	1 01	1.01	1.00	1 00	1.00	0 96	0.96	0.95
30	1 01	1 01	1.01	1.00	1 00	1.00	0 95	0 95	0.95
35	1 01	1 01	1 01	1 00	1 00	1.00	0.95	0.95	0 94
40	1.01	1.01	1.02	1.00	1 00	1.00	0.94	0 94	0.93
45	1 01	1.02	1 02	1.00	1 00	1 00	0.93	0 93	0 91
50	1.01	1.01	1 02	1 00	1 00	1.00	0 93	0.93	0 90
55	1.01	1.02	1 02	1.00	1.00	1 00	0.93	0 92	0 86
60	1.01	1.01	1.03	1 00	1.00	1.00	0 92	0.92	0.81
65	1.01	1.01	1.03	1 00	1 00	0.99	0 87	0 87	0 71
70	1 01	1.01	1 02	1 00	1.00	0.99	0.91	0 91	0 73

Source: See Appendix

Table 9
Work Life Expectancies by Various Smoking Categories, for Men, by Exact Age

	Never Smoked WLE			Lght Smoker WLE			Heavy Smoker WLE		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age	All	Active	Inactive	All	Active	Inactive	All	Active	Inactive
20	36.7	37.2	35.9	35 3	35.8	34.5	34 2	34.7	33 4
25	33 3	33 6	32.0	31 9	32.2	30 8	31 0	31.3	29 5
30	29 1	29 3	27 4	27.9	28.1	26 1	27.1	27 3	25 0
35	24.7	25 0	22.2	23.7	23.8	21 2	23.0	22 8	20 3
40	20.4	20 6	17 2	19.6	19 6	16 4	18 6	18 8	15.5
45	16.0	16.2	12 3	15 3	15.5	11 7	14 4	14 6	10 7
50	11.5	12.1	7 5	11 3	11.7	7 3	10 5	10 8	6 5
55	7.4	8.2	4 1	7 3	8 0	3.9	6.6	7.2	3 3
60	4 1	5.3	2 0	4.0	5 2	1.9	3 6	4 7	1 5
65	1.9	3.9	1.0	1.9	3 8	1.0	1 7	3 3	7
70	9	2 9	5	9	2.9	.5	.8	2 6	4

Source: Each entry is the product of the ratio in Table 7 and the CDG work life expectancy for all men.

Table 10
Work Life Expectancies by Various Smoking Categories, for Women, by Exact Age

	Never Smoked WLE			Light Smoker WLE			Heavy Smoker WLE		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age	All	Active	Inactive	All	Active	Inactive	All	Active	Inactive
20	30.3	30.8	29.4	30 0	30.5	29.1	28.8	29.3	27.9
25	27.0	27.6	25.6	26 7	27 3	25.3	25 6	26.2	24.0
30	23.4	24.1	21.6	23.2	23.9	21 4	22.0	22.7	20.3
35	19 9	20.7	17 7	19 7	20.5	17 5	18.7	19.5	16.5
40	16 3	17.2	13 9	16 1	17 0	13 6	15.1	16.0	12.6
45	12.6	13.8	9.5	12 5	13.5	9.3	11 6	12 6	8 5
50	9.0	10 2	5.6	8 9	10.1	5 5	8.3	9.4	5 0
55	5.7	7 2	2 9	5.6	7.1	2 8	5 2	6.5	2.4
60	2 9	4 6	1 3	2.9	4.6	1.3	2.7	4 2	1.1
65	1 2	3 2	6	1 2	3.2	.6	1 0	2 8	.4
70	5	2.4	3	5	2 4	.3	5	2 2	.2

Source: Each entry is the product of the ratio in Table 8 and the CDG work life expectancy for all women.

Appendix

Calculation of Work Life Expectancies Used to Formulate the Ratios in Table 7 and Table 8 and Work Life Expectancies in Table 9 and Table 10

Ciecka, Donley, and Goldman (CDG) 1992-93 Transition Probabilities

Let ${}^{m}P^{n}_{x}$ denote the transition probability of a person who is in state m at age x being in state n at age $x+1$, where $m\epsilon\{a,i\}$ and $n\epsilon\{a,i,d\}$ and a denotes active, I denotes inactive, and d denotes the death state. We utilize the CDG transition probability data for labor force activity in 1992-93. Equations (1) hold for these data

(1)
$$ {}^{a}p^{a}_{x} + {}^{a}p^{i}_{x} + p^{d}_{x} = 1 $$

$$ {}^{i}p^{a}_{x} + {}^{i}p^{i}_{x} + p^{d}_{x} = 1 $$

where

(2)
$$ {}^{a}p^{d}_{x} = {}^{i}p^{d}_{x} = p^{d}_{x} $$

Hummer, Nam, and Rogers (HNR) 1986 Mortality Probabilities

Let ${}^{ns}p^d_x$, ${}^{ls}p^d_x$, ${}^{hs}p^d_x$, ${}^{t}p^d_x$ and denote the HNR mortality probabilities in Tables 1 and 2 at age x for those who never smoked, for light smokers, for heavy smokers, and for the total population, respectively. These probabilities vary across age groups, but they are constant within the age intervals 25-34, 35-44, 45-54, 55-64, and 65-74. Each of these probabilities is used for the relevant one of the four cohorts (*i e.*, never, light, and heavy smokers and the total population without regard to smoking) for both actives and inactives.

Adjustments to CDG Data to Compute Work Life Expectancies Used to Calculate Ratios in Table 7 and Table 8 and Work Life Expectancies in Table 9 and Table 10.

For ages x = 16, ...,24, the CDG transition probabilities were used in the calculations of work life expectancies. For ages 25,...,75, the HNR mortality probabilities ${}^{ns}p^d_x$, ${}^{ls}p^d_x$, ${}^{hs}p^d_x$ and ${}^{t}p^d_x$ were used in the calculations of work life expectancies for those who never smoked, for light smokers, for heavy smokers, and for the total population, respectively.[6] The non-mortality transition probabilities were re-scaled in the following manner.

(3)

$$\left[\frac{{}^{a}p^{a}_{x}}{{}^{a}p^{a}_{x}+{}^{a}p^{i}_{x}}\right]\left[1-{}^{j}p^{d}_{x}\right] \qquad \left[\frac{{}^{a}p^{i}_{x}}{{}^{a}p^{a}_{x}+{}^{a}p^{i}_{x}}\right]\left[1-{}^{j}p^{d}_{x}\right]$$

$$\left[\frac{{}^{i}p^{a}_{x}}{{}^{i}p^{a}_{x}+{}^{i}p^{i}_{x}}\right]\left[1-{}^{j}p^{d}_{x}\right] \qquad \left[\frac{{}^{i}p^{i}_{x}}{{}^{i}p^{a}_{x}+{}^{i}p^{i}_{x}}\right]\left[1-{}^{j}p^{d}_{x}\right]$$

where $j \in \{ns, ls, hs, t\}$

[6] The HNR mortality probabilities end with the age group 65-74. We simply set mortality probabilities for ages 75-80 (the last age group for which we estimate labor force activity) equal to the mortality probabilities for age group 65-74.

Figure 1

Difference in WLE between Men Who Never Smoked, Light Smokers, Heavy Smokers and WLE Without Regard to Smoking Status

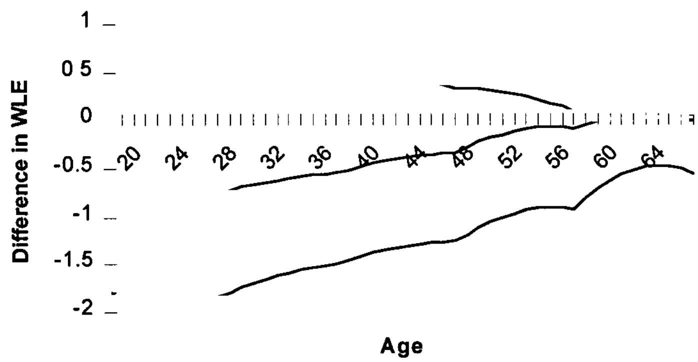
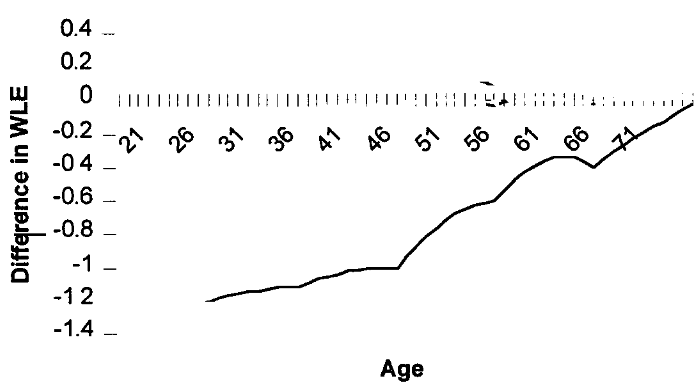


Figure 2

Difference in WLE between Women Who Never Smoked, Light Smokers, Heavy Smokers and WLE Without Regard to Smoking Status



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VALUING THE COMPONENTS OF THE COMPENSATION PACKAGE OF EXECUTIVES

by

David R. Williams*

I. Introduction

This article will provide the practicing economist with guidelines on how to calculate the economic damages in a death, injury, discrimination, termination or any other types of case involving a highly compensated executive. The main theme is to break down the executive's total compensation package into more manageable components and then price each of the components separately.

The total compensation package of a top executive will generally have some combination of the following components: base salary, short-term bonus based on performance over the past twelve months, long-term bonus based on performance over the past 3-5 years, stock options (qualified and/or non-qualified), restricted stock, stock purchase plan and profit sharing. Everything in addition to base salary is not typical of what the average worker receives, and each item should be examined individually. On top of the above there will also be a pension, all the usual insurance (health, dental, disability, life) and special fringe benefits ranging from tax preparation to country club membership. Any practicing economist who has ever dealt with the compensation package of a highly paid executive will have an understanding of the complexities involved in valuation, compared to a normal wage earner.

The definition of what is exactly meant by "executive" is not precise, but generally will cover the vice president level and above. Hence, one would include under this definition chairman, president, chief executive officer (CEO), chief financial officer (CFO), chief operating officer (COO), controller, and general counsel, to name but a few.

There is no typical compensation package for an executive, and research indicates that compensation packages vary quite drastically across companies and industries. However, a clear trend is emerging in that company and individual performance is becoming the leading determinant of executive compensation, particularly at the higher end of the corporate ladder. More and more, bonuses and other incentive pay are being tied directly to individual and company performance measures.

One of the very first steps the practicing economist should do when dealing with a case involving a highly paid executive is to obtain an individualized compensation statement breaking out all the components of the executive's total compensation package. These data should be available from the benefits or human resource department at the company.

If the practicing economist is interested in looking at general historical trends in the growth and size of executive compensation over time, there are four leading sources of

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data for executive compensation in the U.S., namely Business Week, Forbes, William Mercer, Inc. and the Conference Board. Other sources include the RMA Annual Statement Studies, the Corporation Source Book of Statistics of Income, the Almanac of Business and Industrial Financial Ratios, the Officer Compensation Report and the proxy statements of publicly traded companies.

Business Week has conducted an annual survey the past 46 years looking at the compensation of the top two highest paid executives at 362 of the largest U.S. companies. Forbes conducts an annual survey of CEO's total compensation at approximately 800 U.S. companies. William Mercer, Inc. conducts an annual survey of approximately 350 large U.S. corporations looking solely at the compensation of CEOs. The results are published annually in the Wall Street Journal. The Conference Board has an annual analysis of the compensation of the five (it was the three highest until the Securities and Exchange Commission (SEC) changed its reporting requirements in 1992 from three to five) highest paid executives at 270 companies out of a total database of approximately 1,000 companies with sales over $100 million, by major industry.

The precise definition of compensation varies among these four data sources. This data is probably of limited use to the economist on a specific case because it is merely aggregate data. However, it could be useful in helping the economist in establishing overall trends in executive compensation over time.

An additional feature of the Conference Board data is that regression equations are also presented across industry and for each of the five highest paid executives linking executive pay to company size. Company size is measured by different categories across different industries, e.q., total assets for commercial banking and premium income for insurance. The regression equations show that a positive correlation, not surprisingly, exists between company size and executive pay. The regression equations could be used to calculate the average executive pay of an executive in a certain industry by salary rank and by company size.

II. Base Salary and Short-Term Compensation

Base salary can be evaluated using the individual executive's historic salary, the salary of similarly situated executives (if feasible and available) and as a proxy, executive salary by specific industry from such data sources as the Conference Board.

The bonus provided to an executive for performance over the past twelve months is rarely termed a bonus. Corporations are very creative when coming up with names, such as management incentive compensation plan or performance unit incentive plan, for example. In order to project bonuses into the future one can average historical bonus awards or calculate their percent of the base salary. One can also look at the specific firm, the specific industry that the firm is operating in, and the overall market conditions in the economy as a whole to examine whether future cycles are forecast which would affect the pay of the executive in question.

It is very likely that an executive's bonus is formula driven based on company profit, revenue, earnings per share or other company and/or target measures. It could well be a weighted average of various company and individual performance measures relative to set targets. When looking at an executive's historical bonuses one should not be surprised to see some years of zero bonus when targets were not attained by the executive and/or the company.

It is crucial not only to determine a base salary but also its growth into the future. Unfortunately, the use of 1990 Census figures to derive age earnings profiles is not possible because there are no occupational categories for CEO, Chairman, CFO, or any other executive. Future research could be done in this area to determine whether wage increases at the executive level follow the overall trend in other occupations where generally wage increases slow down or peak over the latter stages of the age earnings cycle.

Historic data on the specific individual and industry trends are probably the most important determinants again with regard to the growth rates of salary and bonus. One can also attempt to interview other executives at the corporation to try and ascertain the executive in question's future compensation growth, whether future promotions were planned, and to try and track a "replacement" or similarly situated executive's compensation over time.

III. Longer-Term Compensation

A. Bonus

There can be longer-term bonuses which are based on company and/or individual performance over the past 3-5 years, for example. Many of the comments under bonus and short-term compensation apply again in this section.

B. Stock Options

Stock options are a form of compensation for top executives which has been increasing dramatically in popularity over the past few years. A growing number of companies are now offering stock options to all their employees, not just their executives. A stock option is the right to purchase company stock in the future at a predetermined price (grant price).

The four determinants of the future value of a stock option, discounted to present value, are :

1. the vesting period, which can run anywhere between 1-3 years, with the shares vesting incrementally over the vesting period. Once the stock option is vested it has the potential to be exercised by the stockholder for a gain in value.
2. the life span of the stock option, which generally runs up to 10 years. The stock option is either exercised sometime before the ten years is up or the stock option expires and becomes worthless.
3. the price increase and volatility in the stock. If the stock does not appreciate in price then the stock option is worthless. One can obtain historical price movements in the company stock in question from the internet, brokerage sources or the company itself. For future stock price movements, one can look at the company's own projection of future expected stock price growth from the company's financial reports/newsletters, or make projections based on historical data. One should be careful of such things as past stock price splits, company takeovers diluting the stock price artificially and other nuances when dealing with

a company's stock.

4. discount rate - the interest rate used to discount a future nominal gain in value through the exercising of a stock option for profit back to its equivalent present value at the time of the calculations by the economist.

Simply put a stock option works as follows : say an executive is granted or issued the option or right of purchasing 1,000 shares of his company's stock in the future at today's grant price (which will be today's actual stock price or today's stock price with a discount applied) of $100 per share. The executive cannot take advantage of any increase in the stock price by exercising the stock options and selling these shares for one year, the vesting period in this example. If, after the one year vesting period is up, the stock price has risen to $110 per share, the executive could decide to exercise his stock option by purchasing the stock at the grant price of $100 per share and immediately sell the stock for $110 per share and realize a profit of $10 per share, or $10,000 on all 1,000 shares. The executive also has the choice of not exercising the stock option after one year and instead hoping that the stock will increase in price higher than $110 per share in years two through ten.

The executive cannot lose money with stock options. However, the executive will make no money if (1) the stock price stays at the same level; (2) declines relative to the grant price; or, (3) if the executive does not take advantage of a price increase in the stock which is followed by a price decrease back to the grant price.

There are different types of stock options which fall into two main categories, namely qualified/incentive and non-qualified stock options. Qualified/incentive stock options refer to stock options that are taxed at the time of sale at the capital gains tax rate. Non-qualified stock options refer to stock options that are taxed twice, first at the time of exercise and second at the time of sale. IRS rules govern the tax treatment of different types of stock options.

In terms of the valuation of stock options, there are a number of methods open to the practicing economist. The SEC now requires companies to put a valuation on the unexercised portion of stock options of their five highest paid executives on their company proxy statements. This new SEC reporting requirement took effect in October, 1992. The new SEC reporting requirement allows companies to use the Black-Scholes method of stock option valuation, or other acceptable methods (such as the binomial or the growth model) or to just assume, quite arbitrarily, 5 percent or 10 percent constant compound growth over the full life of the stock option. The appendix goes over the difference between these valuation methods in more detail.

One should also include as an economic loss to an executive stock options which were granted to an executive but lost through the executive's termination because they had not vested yet. Thus, the executive is not in the position to exercise at a future date stock options previously granted.

C. Restricted stock

Companies grant restricted stock to executives, which are forfeitable unless the executive works for the company over a stipulated period of continued employment. Restricted stock is an outright gift of stock given to the executive from the company. The only caveat is that the executive can only sell the stock and realize economic gain once the stock has vested, hence its name restricted. For example, a third of the stock vests each year so that after 3 years all the stock given initially has vested and can be sold by the executive with no restrictions. The vesting period is meant as an incentive for the executive to stay with the current company, and also creates the incentive for the executive to increase the stock price not just for his shareholders, but also for his own personal financial gain. Similarly to stock options, restricted stock can be forfeited due to termination because it had not vested, and should be included under a calculation of economic loss. Concerning restricted stock, the only stock prices that matter are the stock prices at vesting and sale, not the stock price at initial grant. The executive is taxed on the value of the stock when it vests, with many companies inflating or "grossing up" the executive's salary to provide income with which to pay the tax due.

Discounts exist with restricted stock due to a lack of marketability because stock which cannot be readily sold is worth less than stock which can readily be sold. Pratt (1996) summarizes nine studies on this topic and concludes that a discount in the 35 percent to 50 percent range appears reasonable. However, the SEC has recently reduced the rule 144 holding period for restricted stock to one year, down from two years, which means that the discount should now be lower.

D. Stock Purchase Plans

Stock purchase plans, if offered, give an executive the right to subscribe to buy stock in the company at a discounted or preferential price, for example, at a 5 percent-15 percent discount. The subscription price is the discounted price. The subscription price is the maximum price paid for the shares of stock on a given subscription. Unlike stock options and restricted stock, the executive can lose money under a stock purchase plan, if the company's stock price falls below the subscription price in the future.

IV. Other Fringe Benefits

Other fringe benefits available to executives not discussed so far can include the employer's share of social security and medicare, 401(K) - where forfeiture of accrued assets can result from being terminated from the company, profit sharing and all the typical insurance: health, dental, disability (short and long-term), life, and accidental death and dismemberment. Other fringe benefits could also include a company car and associated expenses, educational programs, financial/tax preparation assistance, professional dues, business, health and golf club memberships, flex accounts - to name a few. The practicing economist can use a market approach to obtaining a value on individual fringe benefits and including in the value of the total compensation package. For example, tax preparation can be priced on the open market and converted into a percent of salary.

One should also treat the pension of the executive as a stand alone category under other fringe benefits. Executive pensions are usually of the defined benefit type where the future pension is a function of the number of years worked with the company (usually with an arbitrary percent for each year of service) and the average income over the last or highest three to five years - a formula driven calculation. One needs to be particularly careful in determining what the company defines as "income" in the pension calculation. Base salary is always included, but short or long-term bonuses or other incentive payments may or may not be included under the company's definition of income. It is highly unlikely that stock options, restricted stock and other stock related income would be included under the company's definition of income. Obviously, what is included and excluded under the company's definition of income for the executive's pension will have a tremendous effect on the final pension figure.

Another potentially significant fringe benefit for an executive concerns the benefits derived from a severance package. The practicing economist should check with the attorney to determine whether or not to offset losses by the benefits from a severance package, such as continued health insurance coverage. The economist should also be careful in a reduction in force (RIF) situation where a termination results in an enhanced pension package through a generous severance package. One should also be careful if the pension is reduced by formula for expected social security retirement benefits from the U.S. government. Generally speaking, unless legal considerations override economic logic, one is calculating the difference, in present value terms, between the pension but for the termination and the pension post termination. There may not be a pension post termination due to lack of vesting.

The practicing economist needs to be especially careful in dealing with the fringe benefit package of highly paid executives and should not apply the U.S. Chamber of Commerce fringe benefit percentages of approximately 30 percent-40 percent. Some of the Chamber's percentages will be overestimated and some will be underestimated when looking at highly paid executives. One may end up with an overall percent figure comparable to the Chamber's figure, but for the wrong reasons. If the data permit, one should break out the overall fringe benefit package for the executive into as many individual components as possible and price them out separately. The Chamber's data are more applicable when looking at the rank-and-file employees of a company, rather than the executives at the company. Adjustments have to be made when dealing with executives' benefits. For example, the employer's social security share of FICA was 6.2 percent up to \$62,700 of social security earnings in 1996. However, the appropriate percent to use for an executive making \$500,000 per year is not 6.2 percent, but :

$$0.062 * \$62,700 = \$3,887 / \$500,000 = 0.78 \text{ percent}$$

Similarly, an employer's share of health insurance percent of approximately 4 percent from the U.S. Chamber of Commerce study does not mean that health insurance is constant at 4 percent of income for an employee making \$20,000 per year and a highly paid executive making \$500,000 per year. One would expect this percent to decrease as income rises due to there being an upper limit on the value of a health insurance package. One would expect this to be true for other elements in a fringe benefit package also.

V. Conclusion

This article offers guidelines to the practicing economist when valuing a compensation package of a highly paid executive due to the nuances that generally occur. One should attempt to break out the total compensation package into as many separate components as possible and place a value on each one individually. Issues regarding stock, whether stock options and/or restricted stock, which are generally not an issue in the more typical non-executive case need to be carefully examined and valued appropriately.

More research needs to be done related to the age earnings profile of highly paid executives to see whether pay increases over time follow the same general pattern as for the rest of the workforce. There is no general compensation package for all executives; each one will have its own peculiarities and stock incentives. However, a clear trend has been emerging recently in that executive compensation is becoming increasingly tied to their individual and their company's performance.

Appendix
Stock Option Valuation Methods

A. Black-Scholes Option Pricing Model

This model was developed to value publicly traded call options, but it is also commonly used to value employee stock options. Publicly traded call options differ from employee stock options with regard to a longer time to expiration, delayed exercisability, and non-transferability (i.e. are not traded in a secondary market), to name the most significant differences. These and other differences point to potential shortcomings of using Black-Scholes to value employee stock options, but it is still recognized as producing a reasonable estimate of their economic value. The inputs into the Black-Scholes model are:

1. current stock price
2. exercise price
3. option term
4. interest (discount) rate
5. stock volatility

The formula is :

$$C = S[N(d1)] - Xe^{-Rt} \ [N(d2)]$$

where

$$d1 = \frac{\ln(S/X) + [R + (V/2]t}{(Vt)}$$

$$d2 = d1 - (Vt)$$

C = Option Value
S = Current Stock Price
X = Exercise Price of Option
e = Exponential Constant (2.71828)
R = Interest (Discount) Rate
t = Time Until Option Expires
V = Volatility or Variance in Stock Price
N(d1) & N(d2) =Area Under Standard Normal Distribution, i.e. cumulative normal distribution function

The formula values the difference, in present value terms, between the value of the stock option looking at the current stock price (S) and the exercise price (X), taking into account stock volatility (V). Volatility can be measured using the standard deviation of the log of the daily change in the company's stock price over a historic time period, and should not be confused with the stock's beta. Black-Scholes assumes that the interest rate and stock price volatility are constant throughout the life of the option and that an option can only be exercised at maturity. There are programs for personal computers that will value stock options using the Black-Scholes method readily available, for example OPTION! through Kolb Publishing Company.

B. Binomial Option Pricing Model

This method uses probability theory to estimate the value of an employee stock option. As an example :

- Initial Stock Price	$10
- Probability of Stock Price Going Up	60%
- Probability of Stock Price Going Down	40%
- Magnitude of Stock Price Increase	150%
- Magnitude of Stock Price Decrease	50%

Using the above data, the expected value of the stock after one period is :

$$(60\% * \$10 * 150\%) + (40\% * \$10 * 50\%) = \$11$$

One can construct a lattice diagram over 52 periods for a probability distribution for the stock price at the end of the 52 weeks. The probabilities of the stock price going up or down and the magnitude of the stock price movement up or down are derived from the stock's volatility, which can be calculated based on historic stock price movements.

In contrast to the Black-Scholes method, one can vary the stock price volatility and not have to wait until the end of the term for the stock option to be exercised with the Binomial model.

C. Growth Model Option Pricing Model

This is the simplest of the three methods. One assumes that the stock price grows from the grant price at a fixed compound rate over a given period of time until it is exercised. One calculates the gain in value of the stock options at the exercise date compared to the grant date and discounts this nominal gain in value to present value dollars using an appropriate discount rate.

Other stock option valuation techniques include those of Shelton (1967), Kassouf (1965), and Noreen-Wolfson (1981).

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IMPLICATIONS OF *JOINER*, *CARMICHAEL* AND THE REPORT OF THE JUDICIAL CONFERENCE ADVISORY COMMITTEE ON EVIDENCE RULES FOR FORENSIC ECONOMICS

by

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I. Introduction

On December 15, 1997, the United States Supreme Court issued a much anticipated decision in the matter of *General Electric Co. v. Joiner, 118 S. Ct. 512 (1997).* This was the first time the United States Supreme Court had spoken on the issue of the admissibility of expert witnesses since its very important decision in *Daubert v. Merrell Dow,* 509 US 113 S.Ct. 2786 (1993), 125 L.Ed.2d 469 on June 28, 1993. As this paper is being written, the United States Supreme Court has granted certiorar in another case, *Kumho Tire Company v. Carmichael,* 118 S. Ct 2339 (1998), that may have an even greater impact on economists as expert witnesses. This paper is primarily devoted to an analysis of impact of the *Joiner* decision on forensic economics, but will also explain the importance of the coming *Carmichael* decision. Finally, there is a report of the Advisory Committee on Evidence Rules.

II. What Was Involved in *Joiner*

As in *Daubert,* the *Joiner* decision did not involve an economic expert, but was focused on the admissibility of testimony of experts on the applicability of various medical studies to a particular case. *Joiner* should be seen as clarifying the meaning of *Daubert* and is not likely to have the sweeping impact of *Daubert.* However, *Joiner* reenforces strongly the *Daubert* mandate that federal trial court judges should act as active gatekeepers in determining the quality of scientific testimony permitted in their courtrooms. Justice Breyer's concurring opinion in this matter also points to institutional changes in light of *Daubert* that may have a great impact on the nature of forensic economic practice.

Robert Joiner began work as an electrician in the Water & Light Department of Thomasville, Georgia in 1973. His work involved use of "a mineral-based dielectric fluid as a coolant." This fluid was discovered in 1978 to be contaminated with polychlorinated biphenyls (PCBs). Joiner was diagnosed with small cell lung cancer in 1991 and sued claiming that the PCB's had caused his cancer. At the district court level, Joiner attempted to present experts to testify that the PCBs were the cause of the cancer. The district court ruled that "the testimony of Joiner's experts had failed to show that there was a link between exposure to PCBs and small cell lung cancer. The district court believed that the testimony being proffered did not rise above "subjective belief or unsupported speculation." Joiner then appealed to the 11th Circuit Court of Appeals, which reversed the decision of the district court, setting the stage for an appeal to the United State Supreme Court. The 11th Circuit Court of Appeals based its reversal on a claim that different standards should exist

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for denials of admission of expert testimony than for admissions of testimony.

In its *Joiner* decision, the United States Supreme Court overturned the reversal of the 11[th] Circuit Court of Appeals, thus supporting the district court. There were three parts to the decision. The Court was unanimous with respect to Parts I and II, but with Justice Stevens dissenting on Part III. In addition, Justice Breyer wrote a very important opinion concurring with the majority on all three parts. The United States Supreme Court strongly rejected the notion that different standards should apply when denying admissibility than when allowing an expert to testify. It did so by affirming that the District Court had followed proper procedure, saying:

> Our cases on the subject go back as far as Spring Co. v. Edgar, 99 U. S. 645, 658 (1879) where we said that "cases arise where it is very much a matter of discretion whether to receive or exclude the evidence; but the appellate court will not reverse in such a case, unless the ruling is manifestly erroneous." The Court of Appeals suggested that Daubert somehow altered this general rule in the context of a district court's decision to exclude scientific evidence. But Daubert did not address the standard for appellate review at all. It did hold that the "austere" Frye standard of "general acceptance" had not carried over into the Federal Rules of Evidence. But the opinion also said:
>
> "That the Frye test was displaced by the Rules of Evidence does not mean, however, that the Rules themselves place no limits on the admissibility of purportedly scientific evidence. To the contrary, under the Rules the trial judge must ensure that any and all scientific testimony or evidence admitted is not only relevant, but reliable." 509 U. S., at 589 (footnote omitted).
>
> Thus, where the Federal Rules of Evidence allow district courts to admit a somewhat broader range of scientific testimony than would have been admissible under Frye, they leave in place the "gatekeeper" role of the trial court judge in screening such evidence. A court of appeals applying "abuse of discretion review to such rulings may not categorically distinguish between rulings allowing expert testimony and rulings which disallow it.

Justice Breyer's concurring opinion begins by noting that *Daubert* "will sometimes ask judges to make subtle and sophisticated determinations about scientific methodology and its relation to the conclusions an expert witness seeks to offer." He then adds that "judges are not scientists and do not have the scientific training that can facilitate the making of such decisions." After discussing these points at some length, he adds:

> I therefore want specially to note that, as cases presenting significant science-related issues have increased in number. . .judges have increasingly found in the Rules of Evidence and Civil Procedure ways to help them overcome the inherent difficulty of making determinations about complicated scientific or otherwise technical evidence. Among these techniques are an increased use of Rule 16's pretrial conference authority to narrow the scientific issues in dispute, pretrial hearings where potential experts are subject to examination by the court, and the

appointment of special masters and specially trained law clerks.

Justice Breyer then quotes favorably a passage from an amici brief filed by the *New England Journal of Medicine* that said:

> "[A] judge could better fulfill this gatekeeper function if he or she had help from scientists. Judges should be strongly encouraged to make greater use of their inherent authority . . .; to appoint experts . . .; . Reputable experts could be recommended to courts by established scientific organizations, such as the National Academy of Sciences or the American Association for the Advancement of Science."

The dissent by Justice Stevens agrees with the decision to reverse the 11th Circuit decision, but dissents from the majority decision on the basis of some of the scientific conclusions of the majority. In other words, he agrees that the 11th Circuit decision should be reversed, but objects to majority language that he fears may have scientific implications with which he disagrees. Justice Stevens explains:

> Joiner's experts used a "weight of the evidence" methodology to assess whether Joiner's exposure to transformer fluids promoted his lung cancer [footnote omitted]. They did not suggest that any one study provided adequate support for their conclusions, but instead relied on all of the studies taken together (along with their interviews of Joiner and their review of his medical records . . .)
>
> Unlike the District Court, the Court of Appeals expressly decided that a "weight of the evidence" methodology was scientifically acceptable [footnote omitted]. To this extent, the Court of Appeals' opinion is persuasive. It is not intrinsically "unscientific" for experienced professionals to arrive at a conclusion by weighing all available scientific evidence — this is not the sort of "junk science" with which Daubert was concerned.

The impact of the Joiner decision is to strongly reaffirm that the trial court judge will have considerable discretion in determining whether proffered scientific evidence will meet the requirements of Rule 702 and the mandate of the Daubert decision that federal judges become more active gatekeepers for the admission of scientific evidence. Justice Steven's dissent speaks only to the nature of science and is not a challenge to the discretion of the trial court judge. Justice Breyer's concurring opinion provides the suggestion that trial court judges might more frequently avail themselves of neutral scientific experts as allowed under Rule 706.

All of this points to the need for experts in all fields to be thoroughly prepared with respect to scientific and technical aspects of their methodologies. While the *Frye* standard is simply gone at the federal level, the new *Daubert* standard, while a broader standard, is also a more scientific standard. *Joiner* confirms that the United States Supreme Court confirms that the Court will stand behind District Court judges who become active gatekeepers, even when they are reversed by their own District Courts of Appeal. *Joiner* can be seen as a warning to Courts of Appeal not to interfere with active gate keeping by

District Court judges, which is very much in keeping with *Daubert*.

III. The Relevance of Joiner to Forensic Economics

From the standpoint of a forensic economist, *Joiner* should be understood as a confirmation that the bar that was raised in *Daubert* has not in any way been lowered. An economic expert who goes into a *Daubert* hearing without an adequate grounding in many different levels of economic theory may fail to meet the requirements for admission of that expert's testimony. It is not clear in *Daubert* or *Joiner* whether most economic testimony would fall under the "scientific" or "technical" prongs of Rule 702 of the Federal Rules of Evidence (Johnson and Ireland, 1997), but some of the language in *Joiner* suggests that technical expertise may be held to the same standards as scientific evidence. It appears that weekend trained "economic experts" will not be treated better than "instant experts" in other fields. This, however, is the precise issue to be resolved in the impending *Carmichael* decision, to be discussed below.

Another possible *Joiner* outcome will be that trial court judges will feel newly encouraged to appoint experts of their own to interview the experts offered by each side to determine both qualifications and the scientific merit of their opinions. This particular part of raising the bar for scientific and technical expert testimony may mean a new line of work for experts who can establish a solid record for neutrality in their work for plaintiffs and defense. One would guess that trial court judges will look very carefully at the case records of potential experts in choosing their own experts. An expert with ninety five percent plaintiff testimonies or ninety five percent defense testimonies would not make good candidates for a judge's own expert. At the same time, having been asked to play this role by a judge would significantly enhance the credibility of that witness in the future as an expert for either plaintiffs or defendants. For that reason, Judge Breyer's concurring opinion may very well turn out to be the most significant component of the *Joiner* decision.

IV. What is Involved in *Carmichael*?

The *Carmichael* decision that is on appeal before the United States Supreme Court is also a decision of the 11th Circuit Court of Appeals, cited as Carmichael v. Samyang Tire, Inc., 131 F.3d 1433 (11th Cir. 1997). Kumho Tire Company is one of the co-defendants in that decision. The trial court judge held that the testimony of Dennis Carlson, a purported expert on tire failure, was not admissible under the *Daubert* standards, as not being "scientific." The original examination of the tire whose failure was caused injuries to eight members of the Carmichael family, including one death, was performed by George Edwards. Edwards then became too ill to testify and transferred the case to his employee, Dennis Carlson. Carlson reviewed Edward's file and testified that a design or manufacturing defect in the tire caused the blowout, which in turn caused the injuries to the Carmichael family. The district court judge applied the four pronged *Daubert* test to the proffered testimony of Carlson and excluded that testimony. Since the testimony of Carlson was the only proffered evidence of a tire defect, the trial court judge then granted summary judgement to the defendants.

The case was appealed to the 11th Circuit Court of Appeals on the grounds that a tire expert falls under the classification of either "technical" or "other" in Rule 702, not the "scientific" category. The 11th Circuit Court of Appeals upheld those grounds, using the one test the *Joiner* Court had indicated as a proper basis for a reversal, abuse of discretion by the trial court judge, citing the following colorful language from a similar decision by the 6th Circuit in *Berry v. City of Detroit*, F.3d 1342, 1349-50, (6th Cir. 1994):

> The distinction between scientific and non-scientific expert testimony is a critical one. By way of illustration, if one wanted to explain to a jury how a bumblebee is able to fly, an aeronautical engineer might be a helpful witness. Since flight principles have some universality, the expert could apply general principles to the case of the bumblebee. Conceivably, even if he had never seen a bumblebee, he would still be qualified to testify, as long as he was familiar with its component parts.
>
> On the other hand, if one wanted to prove that bumblebees always take off into the wind, a beekeeper with no scientific training at all would be an acceptable witness if a proper foundation were laid for his conclusions. The foundation would not relate to his formal training, but to his firsthand observations. In other words, the beekeeper does not know any more about flight principles than the jurors, but he has seen a lot more bumblebees than they have.

Many observers believe that the reason the United State Supreme Court agreed to hear the appeal of the decision by the 11th Circuit Court of Appeals is disagreement among the Circuits about whether *Daubert* standards apply to expert testimony that is not "scientific," but either "technical" or "other." The beekeeper in the 6th Circuit decision cited by the 11th Circuit almost surely would fall into the category of "other." The tire expert in *Carmichael* would be more likely to fall into the category of "technical," and thus might well be subject to more restrictions concerning how his testimony is developed than would the beekeeper. A beekeeper might testify from general personal knowledge about the flight patterns of bees, but a tire expert presumably performed some type of examination that could be subjected to technical standards for the purpose of determining the reliability of the procedure that led to the expert's opinion.

In *Daubert*, four tests were proposed, no one of which represented a "necessary" condition for admissibility of scientific testimony, but all of which should be taken into account in determining the admissibility of scientific evidence. Three of the four tests involved issues relating to the testing procedures that were used by an expert in arriving at his opinions: (a) whether the theory espoused by the expert has been tested; (b) whether the theory has been subjected to peer review; and, (c) the theory's known or potential rate of error. The fourth test was simply the old Frye test, relating to (d) acceptability of the procedures used within the scientific discipline. Looked at in the context of a tire expert, the first *Daubert* test may not have been satisfied. The theory that a tire defect can cause a blowout is so non controversial that it may never have been tested. The second test may also not be satisfied. Since it is taken as self evident that a tire defect may case a blowout, it is unlikely that one could find a scientific paper stating that this was so. The third test may also never have been satisfied. The error rate would be the likelihood that a specific defect in the tire would have caused the specific blowout in the case at hand would depend on establishing the exact nature of the defect and its probability of causing the blowout.

Testing the procedure used to arrive at an expert's opinions would involve recreating the same defect in a number of tires and then having a number of test runs of tires with that defect on separate cars to see how often blowouts occurred under similar conditions. However, since such tests would not have been run in the past, the error rate for such tests is unlikely to be known or peer reviewed. All this was evident to the trial court judge, who quite logically concluded that the testimony of the tire expert would not pass the *Daubert* tests. The question before the United States Supreme Court is the same question that was the basis of the 11th Circuit reversal of the trial court judge's opinion: Is the testimony of a tire expert subject to the standards for "scientific" testimony, as defined in *Daubert*, or is it "technical" or "other" and not subject to those same standards?

One observer (Dam, 1998) suggests that the Supreme Court is likely to rule that the tire expert's testimony is non-technical, in which case the Court may do one of three things: (1) The Court can create separate but equal tests for "scientific" and "technical" expert testimony. This would involve perhaps a set of "*Carmichael*" tests for admissibility of technical expert knowledge that focus more on standards for the establishment of technical expertise, which would probably include economic damage analysis, which is inherently untestable in the *Daubert* sense. (2) The Court could abandon the four pronged *Daubert* test in favor of a more general set of tests that could apply to both scientific and technical expert witness testimony. (3) The Court could establish a lower set of standards for technical expertise than is required for scientific testimony. At this stage, it is far from clear what the Supreme Court will do, but it is clear that whatever it does will have enormous consequences for forensic economics. It also seems likely that the bar will be raised for admissibility of economic testimony in at least federal courts.

V. Proposed Changes in the Federal Rules of Evidence

On August 1, 1998, the Judicial Conference Advisory Committee on Evidence Rules (hereafter Advisory Committee) released for public comment proposed amendments to seven evidence rules in the Federal Rules of Evidence (Capra, 1998). These proposals are available for public comment until February 1, 1999, after which the committee will consider whether to change any of its proposals in light of the comments received. After the public comment period, the Standing Committee on Rules of Practice and Procedure will consider whether to send the Advisory Committee's proposed amendments to the Judicial Conference. If the Judicial Conference agrees, the proposals will be sent to the U.S. Supreme Court, which will then consider whether to recommend them to the Congress. If the Supreme Court recommends such changes and the Congress does not act to change them, they would become law on December 1, 2000.

The last time a similar procedure was followed in 1993, forensic economists found themselves needing to keep lists of prior testimonies for the previous four years. In that round of changes, deadlines for providing reports were established and other important changes were made. Among the most important of these proposed changes is the following revised statement of Rule 702:

> If scientific, technical or other specialized knowledge will assist the trier of fact to understand the evidence or to determine a fact in issue, a witness qualified as an expert by knowledge, skill, experience or training, or education, may testify

> thereto in the form of an opinion or otherwise *provided that (1) the testimony is sufficiently based upon reliable facts or data, (2) the testimony is the product of reliable principles and methods, and (3) the witness has applied the principles reliably to the facts of the case.* (Proposed new language in italics).

The Advisory Committee took a strong position in favor of a new unified standard for all expert witness testimony, describing the amendment to Rule 702 as follows:

> The amendment does not distinguish between scientific and other forms of expert testimony. The trial court's gate keeping function applies to testimony by any expert. While the relevant factors for determining reliability will vary from expertise to expertise, the amendment rejects the premise that an expert's testimony should be treated more permissively simply because it is outside the realm of science. An opinion from an expert who is not a scientist should receive the same degree of scrutiny as an opinion from an expert who purports to be a scientist.

The most important change in the proposed language of Rule 702 is likely to be the emphasis on "reliable facts and data." Since the use of discounting formulas and other basic methods of damage analysis are not controversial, it is unlikely that the reliability of principles and methods or how they are applied in a damage analysis will be much of an issue for forensic economics. Using general data sources that have not been tested by peer review may be one practice that will be prohibited, for example. This could apply to the railroad work life tables compiled for the American Railroad Association and the disability work life tales compiled by Vocational Economics, both of which are used by some forensic economists. Other rule changes proposed by the Advisory Committee are less likely to have an impact on forensic economists.

Public hearings on these proposals will be held on October 22, 1998 in Washington, D.C.; on December 4, 1998 in Dallas; and on January 25, 1999 in San Francisco. Written comments can be sent to Peter McCabe, Secretary, Committee on Rules of Practice and Procedure, Washington, D.C. 20544. E-mail comments can be made a www.uscourts.gov. Details on the public hearings can be obtained by calling (202) 273-1820.

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THE STANDARD OF REASONABLE PROBABILITY IN FORENSIC ECONOMICS: A COMMENT

by

John O. Ward

In personal injury litigation economists are asked to provide estimates of damages suffered by a plaintiff with reasonable probability. Economists expressing such opinions seldom qualify them with disclaimers of scientific precision. This observation came home while reading a NAFE-L e-mail query concerning a projection of lost earnings capacity for an injured college student. The issue raised had to do with the appropriateness of assuming that a child would have completed college given the fact that only 20 percent of students in that population completed college. Is the solution to the dilemma to give a range of earnings loss (high school to college), or should the answer be a weighted result [(high school earnings x probability of high school completion) + (college earnings x probability of college completion)]?

The broader question is, do we have a clear idea of what we are doing when we make such forecasts? The issues raised by Daubert (and a number of state courts) is: if the expert is offering an opinion, the expert should be able to state the statistical reliability of the projection. Does the need for such precision relax if we are not considered to be a science? Two types of expert testimony are solicited from experts; scientific and professional. Scientific evidence carries with it a requirement to state the statistical reliability of the scientific projection or opinion (e.q., The probability of this outcome occurring is 95 percent). Weather forecasters would be happy with a 40 percent success rate, but no one really believes them anyway. Medical testimony falls into both camps. A physician who states that a patient has a 75 percent probability of not living beyond five years may be making that statement on the basis of the results of a longitudinal study of such patients by a prominent researcher, or on the basis of their professional experience. A judge may allow such testimony on both grounds.

The standard of reasonable probability has a variety of possible meanings. In a discrimination case we would want to see an inference, whether for or against discrimination, supported by a 95 percent probability statistic. Conversely, in our PI earnings loss projections are we shooting for "more likely than not", "more likely than any other individual outcome" or "a 95 percent probability of accuracy"? Do we even know? The problem is that the courts increasingly want to know. Failure to respond persuasively to the issue of statistical reliability will increasingly result in a dismissal of expert testimony - rightly so!

Stated simply: Assume that you have a 20 year old college sophomore who has suffered an accident resulting in paraplegia. You are asked to tell the jury what present value of earnings and services have been lost, and future medical costs that will likely be incurred. You venture an answer of $2,345,768 based on census earnings and an LPE methodology. If you had 1,000 similar students with similar injuries, how many, do you think would actually suffer losses of $2,345,768 over their lifetimes? Would it be 51 percent, the reasonable probability standard? Would 66 percent of true costs fall within 1 standard deviation of your projection? Would it matter if the person was a male, female, black, white, high income, low income, from the south, rural, urban, smoked, did not smoke? How would you factor each of these data elements into your projection and how would their inclusion affect the probable accuracy of your projections?

Now, we can just say, we are not a science, or we are just an imprecise science. We can say we are a profession and the opinions we express are reasoned opinions that rely upon our experience rather than science. CPA's and psychologists seem to do well here, although psychologists increasingly are being asked to support their opinions with test results (which they do while saying that such tests are worthless). In reality, we are all of these things. We use many of the tools of science and the scientific method (when possible). But, as a social science, you can only stretch the principle of rationality so far. When we are asked to forecast future events, the certainty of our science becomes more suspect.

As economists we are asked to perform several types of tasks in our damages projections. One type involves making assumptions about future career directions. Whether the 20 year old would complete a college degree. The response can be in the form of a probability (assuming it is properly tailored). The other type of task involves, say, the choice of a discount or growth rate, which may be based on an average of an historic period or a spot rate. What is this average and what is its distribution? Can we say anything about a future probability of such a rate existing? Using averages just implies a distribution, 50 percent greater and 50 percent less regarding values. But the events may not be normally distributed, in which case we should use a median (correctly or incorrectly) to arrive at a number that we say is fair or average. It is neither. When you combine a median or mean life expectancy with mean or median historic unemployment rates, participation rates, disability rates, earnings rates, earnings growth rates and discount rates, what do we have as a result - an event whose probability of occurrence is virtually zero? So, it appears that what we wish to impart is a sense of fairness rather than precision. Many of us use ranges of loss, in part, to recognize this imprecision.

I once asked a weather forecaster friend how they came up with such a precise number as a 55 percent probability of rain for a particular day. He responded that they had used a local University Meteorology Department faculty member's model for a while, but now they just come up with a number that sounds reasonable using their experience. Besides, they always have a shot to redeem themselves tomorrow.

Daubert is pushing us to a point of decision. We don't have to live with the precision of our models in the classroom or academic journal. We always have another chance to redeem ourselves next year. In the courtroom, we do act as if we are practioners of science. In the courtroom we are asked for a damages number with the expectation that the number meets a standard of reasonable probability. At best we can say that the formula used to calculate damages was correct and we tried to be conservative and fair in selecting data as inputs into our calculations.

What we do is important. I think that it is largely professional opinion and, a product of our experience. I think we are trying to be fair and as precise as our methods will allow. I don't think we meet the Daubert standard as a science in our projections (with the exception of statistical inference in employment litigation). I think we should give our juries ranges of losses to incorporate our risks of error and admit our deficiencies. Economics is a system of logic based on the scientific method and it uses many tools of science, but our projections of damages seldom meet the standards demanded of a science.

DEFENDING AGAINST A DAUBERT CHALLENGE: A COMMENT

by

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Forensic economic expert witnesses in personal injury and wrongful death cases have had their appearance made more difficult by the Supreme Court ruling in Daubert v. Merrell Dow (Daubert et. al.), individually and as guardians and Litem for Daubert, et. al. V. Merrell Dow Pharmaceuticals, Inc. No. 92-102, argued March 30, 1993, decided June 28, 1993, cited as 113 S. Ct. 2786, 1993) and General Electric Company v. Robert K. Joiner, 118, S. Ct. 512 (1997).

The Court held that the legislatively-enacted Federal Rules of Evidence provide the authority summarizing that relevant evidence is admissible, except as otherwise provided by the Constitution of the United States, by Act of Congress, by these rules, or by other rules prescribed by the Supreme Court pursuant to statutory authority. Evidence which is not relevant is not admissible. Further, relevant evidence is described as that which has any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence. The basic standard of relevance thus becomes a liberal one.

It is noted that Frye predated the Rules by half a century. The pertinence of background common law in interpreting the Rules is recognized and the common law may serve as an aid to their application.

In Joiner, the Court appears to have liberalized the acceptability of expert testimony. It recognizes that it is a matter of discretion by the district courts whether to receive or exclude expert testimony, and that appellate court will not reverse unless the ruling is "manifestly erroneous." The Court noted that although Frye was replaced by the Rules of Evidence, the Rules (especially Rule 702) place the limits in the admissibility of purportedly scientific evidence, nor is the trial judge prevented from screening such evidence. It is the duty of the trial judge to decide which scientific testimony is to be admitted, but also whether it is relevant and reliable.

In Joiner, where the plaintiff was an electrician suffering from lung cancer, the original district court (Northern District of Georgia) excluded testimony on the plaintiff's expert and granted summary judgement. The Eleventh court reversed and certiorari was granted. The U.S. Supreme Court held that the district court did not abuse its discretionary power to exclude scientific testimony in this case because it was based on studies involving infant mice that received massive doses of PCB and other testimony based on epidemiological studies. The appellate court was reversed and the case was remanded to the lower court. In effect, the district court was upheld in that it did not abuse its discretionary power. Courts may find that there it too great an analytical gap between the data and the proffered opinion. The Supreme Court indicated that there were other probable causes for Joiner's cancer.

It is possible that, in the future based on Daubert and Joiner, trial judges will appoint their own experts. This would help scientifically untrained judges and juries with

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court-selected experts, thereby diminishing the influence of biased expert testimony in complicated litigation. This procedure is a common practice in Europe. Acceptability of the testimony of such experts could be determined at pre-trial hearings, thereby decreasing numerous arguments and delays during the course of the trial.